Low Tide in the Bay of Fundy

Above, the Government wharf at Hopewell Cape, at the mouth of the Petitcodiac River, with Folly Point seen in the distance; spring tides drop about 13 feet lower than here shown. Below, a wharf at Kingsport, in Minas Basin, showing the mattress-work supporting vessels at low water.

THE TIDE

BY

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UNITED STATES COAST AND GEODETIC SURVEY

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PREFACE

Almost without exception the published material on tides consists of technical memoirs or treatises in which the subject is developed by means of none too simple mathematics. As a result, an interesting subject is closed to all but the practiced mathematician or specialist in tides.

In this volume an attempt is made to present the subject of tides and more especially the results of recent investigations, without the use of mathematics. The author is indebted to a number of persons for encouragement and suggestions. A list of these would be very largely a register of his colleagues at the United States Coast and Geodetic Survey. He feels, however, that special acknowledgment is due to Col. E. Lester Jones, Director of the Coast and Geodetic Survey, for permission to use, from the wealth of material in the files and publications of the Survey, the illustrative examples presented in this volume, and to Capt. R. L. Faris for continuous advice and encouragement and helpful criticism throughout the preparation of this book. To Dr. W. Bell Dawson, of Ottawa, Canada, until recently Superintendent of the Canadian Tidal and Current Survey, the author is indebted for the illustrative material on the tides in the Bay of Fundy. A further acknowledgment is here made to the editors of the *Geographical Review,* the *Scientific Monthly,* and the *Military Engineer* for their courtesy in allowing the author to republish such portions of the work as have appeared in their pages.

H. A. M.

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THE TIDE

CHAPTER I

INTRODUCTORY

THE sea is never at rest; even in the most sheltered location and in the calmest weather when neither wind nor wave disturb its surface, we find its level, regularly and ceaselessly, to be subject to an alternate rising and falling. This alternate elevation and depression of the surface of the sea, which at most places occurs twice daily, is called the tide. The striking characteristic of the tide is its regularity of occurrence or its periodicity.

To an observer on the coast the tide appears alternately to rise for a period of six hours and to fall for a like period of time. But even a short acquaintance with the tide will bring to light the fact that the amount of rise and fall is not the same from day to day. If we chance to spend some time on a bold rocky coast we will find that the high tide of one day completely submerges rocks which the high tide of another day only partly covers. Likewise the low tide of one day will expose to view rocks which the low tide of another day completely hides from view.

On a gently sloping shore this variation in the extent of rise and fall of the tide from day to day is even more strikingly exhibited. During certain times of the month, high tide will reach inshore farther and cover large areas of low-lying lands, which at other times of the month are

beyond the reach of the tide. It is found, too, that during those days of the month when the tide in its rise reaches farthest, it will, in its fall, recede farthest and at such times expose to view wide reaches of sand or mud flats which at other times of the month are completely hidden by the water at all stages of the tide.

Now it is evident that the great mobility of the water permits heavy winds to raise or lower the normal heights of the tide by considerable amounts. But the dweller along the sea coast knows that while unusual weather conditions may bring about very considerable variations in the height of high or low tide, a much more important variation—more important because it is regular and predictable in its occurrence—is the variation in the height of the tide which is found to be related to the moon's changing phases.

If we note the rise and fall of the tide on the days when the moon is new or full we find that the tide rises higher and falls lower than at other times of the month. And on the days when the moon is in its first and third quarters the tide is found to rise less and fall less than at other times of the month. This has been known for centuries to maritime people and obviously points to some connection between moon and tide.

We may learn something about the tide by idly watching the rise and fall of the water, as it moves ceaselessly in response to the unseen forces that govern the tide. But as in other fields of science, accurate knowledge begins with measurement. Let us therefore measure the rise and fall of the water on a vertical staff graduated to feet and tenths. Such a staff, technically known as a tide staff, may easily be nailed to a pile or other fixed support and we are then in a position to note accurately the height of the tide at any particular time.

In order to observe the tide during the entire twenty-four hours of a day, let us suppose that we had secured

a tide staff to a pile in the harbor of Boston, Massachusetts, and that we were going to record the height of the water on this staff at every half hour for the whole day of April 1, 1922, starting promptly at midnight. But before we begin, it is necessary to note that the heights our tide staff will show are, individually, quite arbitrary. For example, if our tide staff is graduated from 0 to 20 feet, it is evident that at any moment before nailing it to the pile we may make the height of water read anything we please between 0 and 20 feet. In other words, what we are going to record is not the absolute height of the tide at every half hour, but its varying height during the day, as shown by the half-hourly readings.

We want to make sure that our tide staff will be long enough so that high tide will not rise above it nor low tide fall below it. In the harbor of Boston the rise and fall of the tide is, on the average, about 10 feet, though at times it may be as great as 14 feet or more. We will therefore take a staff graduated from 0 to 15 feet and nail it to a pile so that the 15-foot mark is somewhat above the highest watermark on the pile.

Let us now assume that on March 31, 1922, just before midnight, having secured our tide staff to a pile, and with pencil and paper at hand, we are ready to begin our tide observations. Promptly at the stroke of the clock we look at the tide staff and find that the surface of the water coincides with the 11.1-foot mark; this we enter into our record as the zero hour reading. Half an hour after midnight, to the nearest minute, we make another observation and find the height of the water on our staff to read 11.6 feet, which we again record; and so we continue throughout the day, recording the height of the tide every thirty minutes and ending with the stroke of midnight that ends April 1st. As 24 hours is a considerable stretch of time for one person we had arranged for relays dividing the

day into three shifts—0 to 8:00 A.M., 8:00 A.M. to 4:00 P.M., 4:00 P.M. to midnight.

We are now in possession of a set of 48 figures, showing the height of the tide in Boston Harbor, with reference to our tide staff, for each hour and half hour of April 1, 1922. But figures alone do not help us greatly in visualizing the phenomena of the tides. We will therefore resort to the

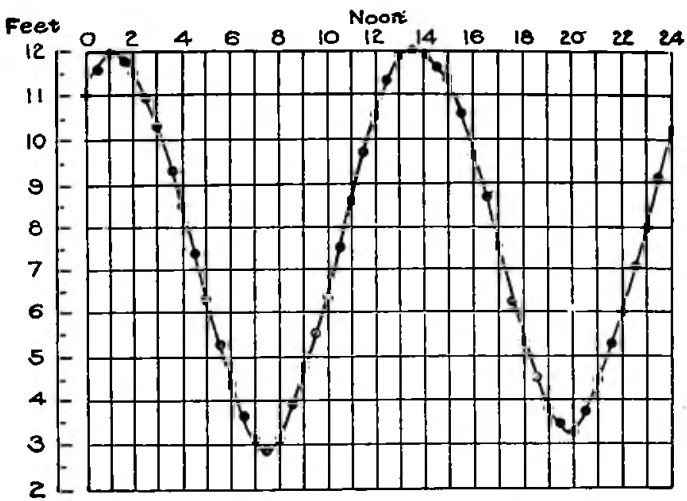

FIG. 1.—TIDE CURVE, BOSTON HARBOR, APRIL 1, 1922

device of a diagram, or what the technically inclined are pleased to call a graph. This is easily made with the aid of paper ruled horizontally and vertically, as shown in Figure 1.

In the diagram above, the figures from 0 to 24, increasing from left to right, represent the hours of the day, beginning with midnight. Numbering the hours continuously to 24 has, for our purpose, many advantages, not

the least of which is that all uncertainty as to whether morning or afternoon is meant is eliminated—6:00 o'clock means definitely 6 in the morning, and 18:00 o'clock as definitely means 6 in the evening. The figures on the left, increasing upward from 2 to 12, represent the height of the tide, in feet, as measured on our fixed vertical tide staff. The dots thus represent the height of the tide at each hour and half hour, and, if through these points we draw a continuous curve, we have pictured before us the rise and fall of the water in Boston Harbor throughout the entire day.

The characteristics of the tide are most advantageously studied from a tide curve constructed as outlined above. But before taking up the consideration of the tide curve for Boston, represented in Figure 1, it will be of advantage to define carefully a few terms in general use.

As ordinarily used, the terms high tide and low tide are not as explicit in their meaning as might be desired. Thus, while high tide is generally correctly used to designate that phase of the tide when the rise has ceased and the fall is about to begin, it is also used at times to designate high stages of water irrespective of the phase of the tide. For the sake of exactness, therefore, the terms high water and low water are used to designate respectively and highest point of the rising tide and the lowest point of the falling tide. Instead of high tide and low tide we shall therefore use, hereafter, the terms high water and low water.

The "range of tide" is another term of frequent use in tidal discussions. This term fortunately has no ambiguity attached to it. By range of tide, we designate the difference in height between high water and low water. If it is the average difference in height between high water and low water that we have in mind, we speak of the mean range or the average range. Other adjectives are used to

designate the ranges at particular times, as we shall learn later; but in every case the range of tide is the difference in height between high water and low water.

A glance at Figure 1 is sufficient to show that in its rise to high water and in its fall to low water the tide does not move at a uniform rate. Thus, from 1:00 o'clock, when high water occurred and the tide began its fall, the motion was very slow at first; gradually it increased until at about 4:00 o'clock it was falling fastest. After that time the rate of fall began diminishing, so that near the time of low water the tide was falling very slowly. At 7:30 the fall stopped, low water occurred, and a similar cycle in the rising direction began. To those familiar with mathematics the curve of the tide presents a well-known form, approximating closely the curve known as the cosine curve.

The tide curve further shows that on the day represented there were two high waters and two low waters in Boston Harbor. The high waters on that day reached just about the same height, the morning high water measuring 12 feet on our staff and the afternoon high water 12.1 feet. The two low waters differed somewhat, the first one measuring 2.8 feet and the second one 3.2 feet. The range of the tide for that day was therefore 9.0 feet.

The tide curve, besides showing the height of the tide at any desired time and thus visualizing the phenomenon for us, has the further advantage in that the times of high and low water may be easily determined. Thus from our tide curve for Boston, we see that the first high water occurred at 1:00 and the second at 13:30 (1:30 P.M.). The first low water came about 7:30 and the second about 19:50. It follows therefore that the difference in time from one high water to the next is about 12½ hours, and that this is also the interval between two low waters. High water and low water are seen to follow each other in a very nearly

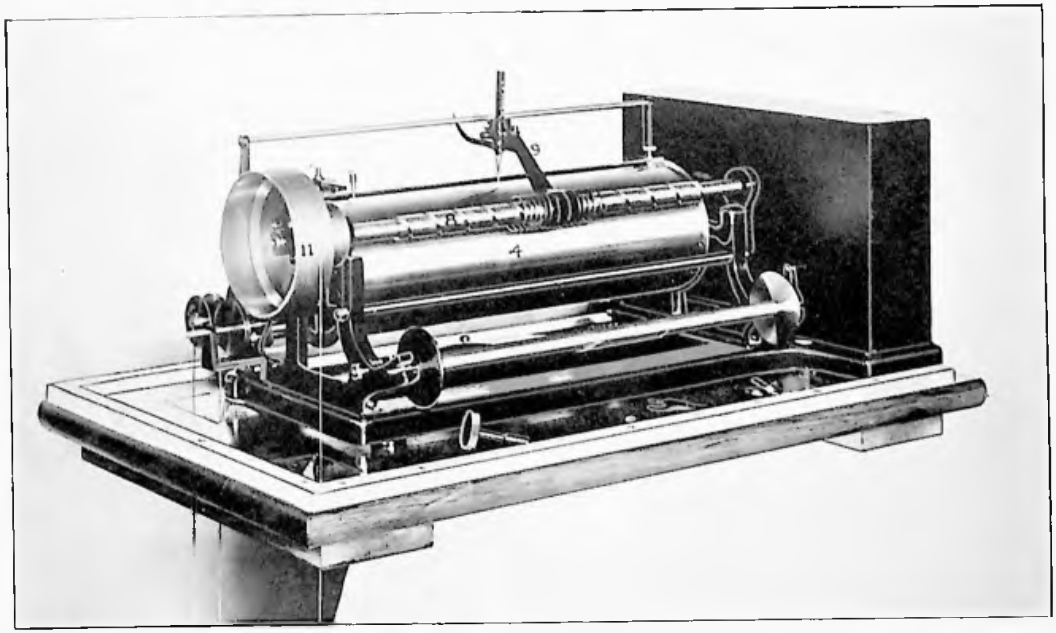

FIG. 2. AUTOMATIC TIDE GAUGE

designate the ranges at particular times, as we shall learn later; but in every case the range of tide is the difference in height between high water and low water.

A glance at Figure 1 is sufficient to show that in its rise to high water and in its fall to low water the tide does not move at a uniform rate. Thus, from 1:00 o'clock, when high water occurred and the tide began its fall, the motion was very slow at first; gradually it increased until at about 4:00 o'clock it was falling fastest. After that time the rate of fall began diminishing, so that near the time of low water the tide was falling very slowly. At 7:30 the fall stopped, low water occurred, and a similar cycle in the rising direction began. To those familiar with mathematics the curve of the tide presents a well-known form, approximating closely the curve known as the cosine curve.

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uniform manner, the difference from one to the other being a little over 6 hours.

If we observed the rise and fall of the tide in Boston Harbor for a number of days we would find the features noted above to be characteristic features of the tide. Every day high water and low water would succeed each other at intervals of a little over 6 hours. While these intervals would vary somewhat, being sometimes less than 6 hours and sometimes more, the average interval would be found to be 6 hours 12 minutes. Striking differences, however, would appear as between the heights of high water of succeeding days and likewise between the heights of low water.

Even for one day, the recording of the tide at frequent intervals throughout the 24 hours is something of a task if done by human labor. But it is a very simple matter to devise a mechanical means to record automatically the rise and fall of the tide. In principle all we need do is to take some object that will float in the water and connect it by suitable means with a pencil which moves up and down a sheet of paper that is driven forward at a uniform rate. A contrivance of this sort is known as an automatic tide gauge and various forms are in use. Figure 2 represents the type used by the U. S. Coast and Geodetic Survey and is designed to record the tides continuously on a roll of paper for the period of a month.

The float, which is not shown in the figure, is connected with the large pulley (No. 11) by means of a wire, so that, as the float is carried upward and downward by the tide, its motion is transmitted through the wire to the large pulley which in turn is made to move the pencil carrier (No. 9). This moves the pencil over a sheet of paper that is carried forward uniformly over the large cylinder (No. 4). Two clocks contained in the case shown to the right of the figure furnish the motive power, one driving the

paper-feeding mechanism and the other marking the hours on the record.

Obviously it would be impractical to have the rise and fall of the tide recorded, on the paper, in its natural scale. The length of the cylinder over which the paper feeds (No. 4) is about 13 inches. Hence, it is necessary to introduce some means for reducing the rise and fall of the tide so that it might be accommodated on a roll of paper 13 inches wide. This is done by means of the pencil-carriage screw (No. 8), the relation of the pitch of the screw to the diameter of the float-wire pulley (No. 11) determining the scale of reduction.

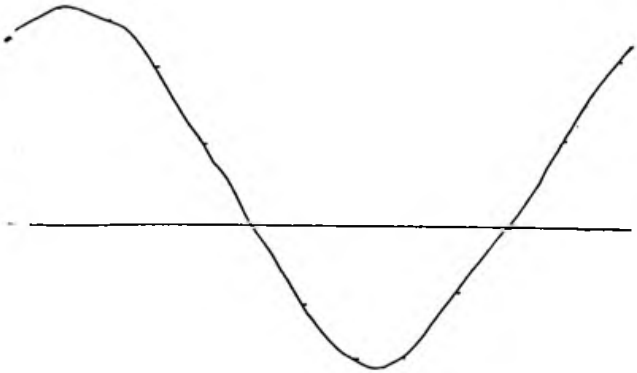


FIG. 3.—TIDE GAUGE RECORD (REDUCED SCALE), BOSTON HARBOR, FORENOON OF APRIL 1, 1922

The tide gauge represented in Figure 2 requires little attention, and, barring accidents, the winding of the clocks once a week is all the labor involved. The paper is driven forward at the rate of 1 inch per hour so that a roll of paper less than 2 inches in diameter accommodates a month of record. A specimen curve for the forenoon of April 1, 1922, made by this form of tide gauge in Boston Harbor is shown on a reduced scale in Figure 3. The short hori-

zontal marks on the curve are hour marks made by the time clock and indicate the beginning of each hour. The long horizontal line in the middle of the diagram is the base line to which the height of the tide is referred.

The tide curve made by the automatic tide gauge possesses all the advantages of the tide curve made by hand and has further the great advantage of being secured at little cost. Suppose now that we are in possession of an automatic tide gauge record for the harbor of Boston for the whole month of April, 1922. If we tabulate the average height of high water for each day and also the average height of low water, we would find that these heights vary throughout the month. But these variations, notwithstanding disturbing effects of wind and weather, are not irregular and haphazard; they exhibit marked periodic characteristics. And on closer examination we should find them going through cycles of increasing heights and decreasing heights, the periods of each of these cycles being about seven or eight days.

Now the striking feature of these periodic cycles is that they are related to the phases of the moon. This is brought out graphically in Figure 4 in which the average heights of high water and of low water for each day of the month of April, 1922, and also the phases of the moon for that month, are plotted.

The diagram immediately brings to light the fact that the tide rises highest about the times of full moon and new moon, and rises least about the times of the moon's first and third quarters. The height of the low waters is likewise seen to be related to the moon's phases, falling most near the times of full moon and new moon and least when the moon is in its first and third quarters.

The tides that occur when the moon is full or new are called spring tides and the range of the tide at such times is known as the spring range. Since the tide rises highest

and falls lowest when the moon is full or new, it follows that the spring range of the tide is greater than the average range. The tides that occur when the moon is in its first and third quarters are known as neap tides, and the range of the tide at such times is called the neap range. Since high water does not rise as high at the times of the moon's first and third quarters as at other times of the month, nor low water fall as low, it follows that the neap range of the tide is less than the average range.

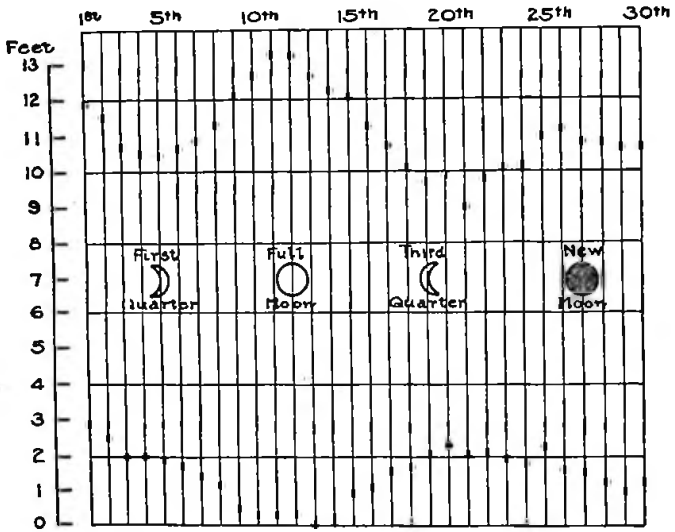


FIG. 4.—HEIGHTS OF HIGH AND LOW WATERS, AND MOON'S PHASES, BOSTON HARBOR, APRIL, 1922

The rise and fall of the tide varies greatly over the world. In Boston, the mean range of the tide is very nearly 10 feet; on Nantucket Island, less than 100 miles south, it is just a little over one foot and in the Bay of Fundy, about 400 miles northeast of Boston, it is over 40 feet. We shall

find further that, besides differences in range, there are also other differences in the tides at various places. But the relationship we found existing between moon and tide in Boston Harbor is characteristic of the tide the world over with but few exceptions. However, it must be added that the greatest rise and fall does not come exactly on the days of new and full moon. There is generally an interval of one or two days between full moon or new moon and the greatest range of the tide. And a like interval is found between the first and third quarters of the moon and the smallest tides.

The *height* of the tide, then, is in some way related to the changing phases of the moon. Is there any relation between the moon and the *time* of the tide? To answer this it will be necessary to examine the times of high water and low water at some place over a period of several weeks. We might, of course, use the tides at Boston for April, 1922; but some reader might suspect a special affinity of the moon for the Boston tides or an unusual state of tidal affairs during the month of April, 1922. Let us therefore choose some other place and time. Let us take the tides in the harbor of New York for the month of September, 1917. Here, too, we have the benefit of the tide record furnished by an automatic tide gauge.

In New York Harbor, at Fort Hamilton, on September 1, 1917, the morning high water occurred at 7:25; the following day it was 8:05 when high water occurred, and on the 3rd it was exactly 9:00 o'clock. This retardation in the time of high water continued throughout the month, the time on the 5th being 10:35 and on the 11th 16:20 (4:20 P.M.). Hence from the 1st to the 11th—10 days—the time of high water was retarded 8 hours 55 minutes, being an average daily retardation of a little over 50 minutes.

A similar retardation of about 50 minutes characterizes

the afternoon high water of the day, and also the morning and afternoon low waters. Sometimes the tide of one day follows the tide of the preceding day by a little more than 50 minutes and at other times by a little less; but the average period is almost exactly 50 minutes, this period being characteristic of the tide not only in New York, but practically everywhere. And the interesting feature of this particular interval is that the moon in its revolution around the earth shows the same retardation. From day to day, on the average, the moon is 50 minutes later in its passage over the meridian of any place. Hence, the time of tide, like the height, appears to be related to the movement of the moon.

Since the tide and moon are both retarded in their appearance by the same interval, it follows that the relation of the moon's meridian passage to the time of high or low water at any place should be constant or very nearly so. And this we find to be the case. In New York Harbor, at Fort Hamilton, high water is found to come about 7¾ hours and low water about 1½ hours after the moon's transit. These intervals will vary somewhat through the month, but not greatly. Whenever at Fort Hamilton we see the moon over the meridian we may be sure that 1½ hours later the tide will be at low water or very nearly so, and likewise that 7¾ hours after the transit, the tide will be at high water or very close to it.

The phenomena of the tides, as we shall find, present numerous features which vary from place to place. The features discussed in the preceding introductory remarks are common to the tides the world over, with but few exceptions, and are therefore called the characteristic features of the tide. These may be summarized as follows: In its rise and fall the tide does not move uniformly, but in a manner represented closely by the cosine curve. High water and low water succeed each other by average intervals

of 6 hours 12 minutes, thus generally giving two high waters and two low waters in a day. The heights of high and low water are found to vary in accordance with the moon's changing phases, the tide rising higher and falling lower when the moon is full and new and rising least and falling least when the moon is in its first and third quarters. The time of tide, too, is related to the movement of the moon, as shown by the average daily retardation of 50 minutes in the times of high and low water and by the very nearly constant interval between these times and the moon's meridian passage.

CHAPTER II

THE DEVELOPMENT OF TIDAL KNOWLEDGE

THE tide is a phenomenon of everyday occurrence; it is therefore reasonable to suppose that the regular rise and fall of the sea must have been known for ages to that portion of mankind that lived close to the sea. It so happens, however, that the peoples whose history has come down to us and whose thought and knowledge have influenced us most—the Greeks, the Romans, and the Hebrews—lived on or near the shores of the Mediterranean where the tide is very small. As a consequence, the tide received scant attention from these people, since it was of little importance in their everyday affairs; and so the tidal knowledge possessed by them was not very extensive.

In consequence of the small range of the tide in the Mediterranean, the tidal phenomena were not very impressive; furthermore, the regularity of occurrence of the tides was frequently masked by the disturbing effects of wind and atmospheric pressure. Hence, not only did the maritime people of antiquity disregard the tides in their everyday affairs, but even as a subject of study and speculation tidal phenomena received little attention. So far as Biblical literature is concerned, there appears to be no direct reference to the tide either in the Old or New Testaments. And even in the classical literature of the Greeks and the Romans the passages dealing with the tides are relatively few in number.

The earliest reference to the tide so far as known appears

in the writings of the early Greek historian, Herodotus, who was born about 485 years before the beginning of the Christian era. In describing a certain arm of the Red Sea, he says:

> Now in Arabia, not far from Egypt, there is a gulf of the sea entering in from the sea called Red. . . . Every day the tide ebbs and flows therein.

In common with the early explanations advanced for other physical phenomena, the earlier attempts at explaining the tides appear to have been based largely on fanciful notions. Thus, some of the ancient philosophers believed the earth to be an animal; it therefore appeared entirely logical to ascribe the rise and fall of the tide to the breathing of this earth animal or to its drinking in and spouting out a certain portion of the water. Another explanation, based on the same belief, regarded the water as constituting the blood of the earth and the tide as the beating of its pulse.

With the development of the knowledge of the physical world and with the growth of a more critical spirit, more rational theories were advanced, and we find the tide ascribed to differences in the level of the sea, to the discharge of rivers into the sea, to whirlpools and eddies, and, finally, to sun and moon. Just how early the connection between moon and tide had been recognized we do not know; we do, however, have record of the fact that Pytheas of Massilia, who lived about 325 B.C. and who navigated the ocean from the Strait of Gibraltar to the British Isles, noted the relationship existing between the tide and the moon.

When we come to the first century of the Christian era, we find the tides definitely ascribed to the action of sun and moon. The following quotations from Pliny's *Natural History,* which appeared in Latin in the year 77 A.D.,

exemplify the state of knowledge regarding the tides at that time.

Much has been said about the nature of waters; but the most wonderful circumstance is the alternate flowing and ebbing of the tides, which exists, indeed, under various forms, but is caused by the sun and the moon. The tide flows twice and ebbs twice between each two risings of the moon. . . .

There is a difference in the tides, depending on the moon, of a complicated nature, and first, as to the period of seven days. For the tides are of moderate height from the new moon to the first quarter; from this time they increase, and are the highest at the full: they then decrease. . . .

All the seas are purified at the full moon; some also at stated periods. . . . To what has been said above (not to omit anything with which I am acquainted) Aristotle adds, that no animal dies except when the tide is ebbing. The observation has been often made on the ocean of Gaul: but it has only been found true with respect to man.

In the chapters devoted to the tides, from which the above quotations were taken, Pliny describes some of the principal phenomena of the tides. He is aware that there are two high and two low waters in a day, that the tides at any given place follow the moon's meridian passage by an approximately constant interval, that the extent of rise and fall varies with the changing phases of the moon, and that the tides are higher at the times of the sun's equinoxes than at the solstices. He does not, however, advance any theory to explain the connection between moon and tide, except that in the conclusion of the section devoted to the tides he says:

Hence we may certainly conjecture that the moon is not unjustly regarded as the star of our life. This it is that replenishes the earth; when she approaches it, she fills all bodies, while, when she recedes, she empties them. From this cause it is that shellfish grow with her increase, and that those animals which are

without blood more particularly experience her influence; also, that the blood of man is increased or diminished in proportion to the quantity of her light; also that the leaves and vegetables generally, as I shall describe in the proper place, feel her influence, her power penetrating all things.

In the First Century A.D., we therefore find it definitely established that the tide is brought about by the combined action of sun and moon, and that the moon plays the leading rôle. How the sun and moon bring about the tide was not understood, nor why the moon should be the controlling factor.

Some attention appears to have been given to the tides in the centuries following. And on the coast of England, where the rise and fall of the tide is very much greater than on the shores of the Mediterranean, we find the noted English scholar, the Venerable Bede, devoting to the tides a chapter of one of his scientific treatises. Writing at the beginning of the Eighth Century in Latin he says, under the title "On the bond between the sea and the moon":

Most to be wondered at above all things is the close tie existing between the ocean and the moon . . . just as if it were unwillingly dragged along by some influence of the moon and again, when the moon's force has ceased, was poured back into its own proper basin. . . .

So also the sea follows the course of the moon not only in its ordinary coming and going, but also in a continual waxing and waning; not only is it later than the day before, but the tide also returns greater or smaller as the case may be. . . .

So let him, who can, see whether it is true what some say and attempt to prove, that the great pouring forth of the ocean which must go up the rivers of all regions, comes to pass at the same moment! For we know, we who live in different parts along the coast of the British Sea, that where in one place the sea begins to flow, at the same time in another place it begins to ebb. . . . Besides in other countries an ebb of the sea is caused by the moon in that part of the sky in which here it causes a flow; also on the same shore where folks are living to

the north of me every tide is received and poured back again much earlier than here, while to the south the tide is much later, since in every country the moon keeps ever the rule of alliance with the sea which it once for all has agreed upon.

It is to be remembered that Bede was familiar with the Latin and Greek authors and availed himself of their writings. And the tide, too, is a much more impressive phenomenon on the coast of England than on the shores of the Mediterranean. Yet there appears to have been very little increase in tidal knowledge, and no advance toward a solution of why the moon and sun cause the tide, in the more than six hundred years separating Pliny and Bede.

In the Thirteenth Century an Arabic author, Zakariyyā ibn Muhammad ibn Mahmūd al Qazvīnī, devotes some space to the tides in a book entitled *Wonders of Creation.* This author attempts a physical explanation of the action of sun and moon in bringing about the tide, as the following quotations show:

Know that at different periods of the four seasons, and on the first and last days of the months, and at certain hours of the night and day, the seas have certain conditions as to the rising of their waters and the flow and agitation thereof.

As to the rising of the waters, it is supposed that when the sun acts on them it rarefies them and they expand and seek a space ampler than that wherein they were before, and the one part repels the other in the five directions eastwards, westwards, southwards, northwards and upwards, and there arise at the same time various winds on the shores of the sea. This is what is said as to the cause of the rising of the waters.

As for the flow of certain seas at the time of the rising of the moon, it is supposed that at the bottom of such seas there are solid rocks and hard stones, and that when the moon rises over the surface of such a sea, its penetrating rays reach these rocks and stones which are at the bottom, and are then reflected back thence; and the waters are heated and rarefied and seek an

ampler space and roll in waves towards the seashore . . . and so it continues as long as the moon shines in mid-heaven. But when she begins to decline, the boiling of the water ceases, and the particles cool and become dense and return to their state of rest, and the currents run according to their wont. This goes on until the moon reaches the western horizon, when the flow begins again, as it did when the moon was in the eastern horizon. And this flow continues until the moon is at the middle of the sky below the horizon when it ceases. Then when the moon comes upward, the flow begins again until she reaches the eastern horizon. This is the account of the flow and ebb of the sea. . . .

And this the Prophet (on whom be the blessings of God and His peace) hath expressed in a poetical manner, when he says: "Verily the Angel, who is set over the seas, places his foot in the sea and thence comes the flow; then he raises it and thence comes the ebb." [1]

Although written about twelve hundred years after the appearance of Pliny's *Natural History*, we find in the above quotations from the *Wonders of Creation* little progress toward the solution of the problem of the tides. In fact, little progress can be recorded till the latter half of the Seventeenth Century. While we find some philosophers, Kepler among them, attributing the tides to an attractive force of the moon partaking of the nature of magnetic attraction, there were not wanting others, among whom must be mentioned Galileo, who contended that the idea of the moon being the principal cause of the tide was preposterous. The state of knowledge regarding the tides, prior to Newton's discovery of the law of gravitation, is well summarized in the following quotation from the *Geography* of Bernhardus Varenius, which appeared in Latin in 1650:

There is no Phænomenon in Nature that has so much exercised and puzzled the Wits of Philosophers and learned Men as this.

[1] Quoted from Darwin's *The Tides and Kindred Phenomena in the Solar System.*

Some have thought the Earth and Sea to be a living Creature, which by its Respiration, causeth this ebbing and flowing. Others imagined that it proceeds, and is provoked, from a great Whirlpool near Norway, which, for Six Hours, absorbs the Water, and afterwards, disgorges it in the same space of Time. Scaliger, and others, supposed that it is caused by the opposite Shores, especially of America, whereby the general Motion of the Sea is obstructed and reverberated. But most Philosophers, who have observed the Harmony that these Tides have with the Moon, have given their Opinion, that they are entirely owing to the Influence of that Luminary. But the Question is, what is this Influence? To which they only answer, that it is an occult Quality, or Sympathy, whereby the Moon attracts all moist Bodies. But these are only Words, and signify no more than that the Moon does it by some means or other, but they do not know how: Which is the Thing we want.[2]

In 1687, Isaac Newton published his *Principia,* and here, for the first time, the connection between moon and tide received a rational explanation. Newton proved that the tides were a necessary consequence of the law of gravitation, which states that two bodies attract each other with a force directly proportional to their masses and inversely proportional to the square of the distance between them. The sun and moon in their varying positions relative to the earth bring about attractions which are different as regards the solid earth and the overlying waters, and it is these differences of attraction which give rise to the tides.

Newton simplified the problem by supposing the sea to cover the whole earth and to assume at each instant a surface of equilibrium; that is, he treated the problem as one of water at rest or, in the words of the mathematician, as a static problem. By this method of treatment, he was able to deduce the principal phenomena of the tides. And after showing that the theory of gravitation was adequate

[2] From an English translation quoted in Harris' *Manual of Tides.*

to account for the tide, Newton did not push his investigations further, leaving the development of the theory of the tides to subsequent investigators.

Newton's theory of the tide, known as the Equilibrium Theory, furnished the foundation on which all subsequent tidal work is based. But it soon became evident that in its simple form the Equilibrium Theory could not be made to explain the characteristics of the tide at numerous places. And with increased knowledge of the times and heights of the tide the world over, it was found that it failed utterly in giving a comprehensive view of the time and height of the tide as it occurs in the seven seas. In other words, while Newton's theory showed that the gravitational attraction of sun and moon is adequate to explain the principal phenomena of the tide, it did not explain the differences in time, in range, and in the characteristics of the tide as they are actually found in nature.

In 1738—about fifty years after the formulation of the law of gravitation by Newton—the Academy of Sciences at Paris proposed the problem of the tide as the subject of a prize essay. Two years later this prize was divided among four contestants: Daniel Bernoulli, professor of anatomy and botany at Basel; Colin Maclaurin, professor of mathematics at Edinburgh; Leonard Euler, professor of mathematics at St. Petersburg, and the Jesuit, Antoine Cavalleri. The first three mentioned based their essays on the principle of gravitation and on Newton's theory of the tide, but Cavalleri based his on Descartes' theory of vortices which had enjoyed considerable popularity the century before.

It is to be noted that in attacking the problem of the tide, the aim of the mathematician has been to derive a general formula which would express completely the rise and fall of the tide at all points of the earth's surface; that is, a formula which, for any place and time desired, would

give the height of the tide by substituting in it such factors as the latitude and longitude of the place and the latitudes and longitudes of the sun and moon at that time. This the equilibrium theory did not succeed in doing, but in the hands of Bernoulli it was developed sufficiently to enable predictions of the time and height of high and low water at any given port to be made, when based on a long series of tidal observations at that port.

Toward the end of the Eighteenth Century a very considerable advance in the theory of the tide was made by the great French mathematician and astronomer, Laplace. Instead of treating the problem as one of equilibrium, as Newton and his successors up to that time had done, Laplace attacked the problem of the tide as one of water in motion, that is, he approached it from the viewpoint of dynamics and his theory is known as the Dynamic Theory of the tide.

Laplace's theory of the tide is contained in his great work on celestial mechanics, the *Mécanique Céleste,* and it has been of profound importance in the mathematical development of the subject. But the solutions of the equations resulting from the dynamic theory, after introducing the complex conditions of the existing oceans, either surpass the power of mathematics or necessitate such enormous labor as to be practically impossible; so that Laplace's theory, although very profound, does not succeed in expressing by means of a general formula the rise and fall of the tide as we actually find it in nature.

A different approach to the solution of the problem of the tide as one of water in motion was made in 1842 by the English mathematical astronomer Airy, who treated the rise and fall of the tide as the movement of waves in canals. While expressly stating that this theory was imperfect, since it could not be applied to every part of the ocean, he nevertheless derived a number of important results

which serve to explain many of the observed phenomena of the tides in rivers and channels.

Following Airy, a number of eminent mathematicians have added to the further development of the theory of the tides. As outstanding figures may be mentioned Ferrel and Harris in America; Kelvin, G. H. Darwin, Lamb, and Hough in Great Britain; Lévy and Poincaré in France; Börgen in Germany; Van der Stok in Holland. At the same time, too, numerous observations on the tides in the various oceans had been made, increasing enormously our knowledge of the geographic distribution and local peculiarities of the tides.

This increase in knowledge of the tides in the seven seas was brought about, in part, by the need for accurate tidal information in connection with the maintenance of exact schedules in the sailing of the deep-draught vessels of modern commerce, and in part, too, by the invention of the self-recording tide gauge and the development of more powerful mathematical methods for analyzing tidal observations. These matters will receive further mention in succeeding chapters.

CHAPTER III

THE TIDE-PRODUCING FORCES

For many centuries mankind had been familiar with the existence of some bond between moon and tide. But the precise nature of this bond, or rather the agency by means of which the moon exerted its influence on the waters of the earth, remained an unsolved problem until the latter part of the Seventeenth Century. It is to the genius of Sir Isaac Newton, the great English mathematician, that we owe the first rational explanation of the relation between moon and tide.

The mathematician, by the use of equations involving the masses and distances of sun, moon, and earth, has no difficulty in computing the tidal forces for any point on the surface of the earth. And it must be admitted that it is very much easier to explain the forces mathematically than to do so without mathematics. We shall therefore content ourselves with showing that the attraction of sun and moon bring about forces which cause the waters of the oceans to move relative to the solid earth and that the phenomena of the tides find explanation in these forces. It will simplify matters to consider the action of sun and moon separately and to assume the earth entirely covered with a layer of water.

In Figure 5 let the circle with center at E represent the earth, the surface of which is entirely covered with a layer of water, and let M represent the moon. Taking any point on the ocean, as P, let us see what forces are brought into play as the position of point P in relation to the moon

changes, due to the rotation of the earth and the revolution of the moon. It will further simplify matters if we assume the moon to be moving in the plane of the equator and if the point P is taken on the earth's equator.

When our point is directly beneath the moon, as in position P_1, it is evidently nearer to the moon than the solid earth as a whole; for the average distance of the moon to the solid earth is measured by the line from M to E, which is greater than the distance from M to P. The intensity, therefore, with which the moon attracts the water at P_1, is greater than its intensity of attraction for the

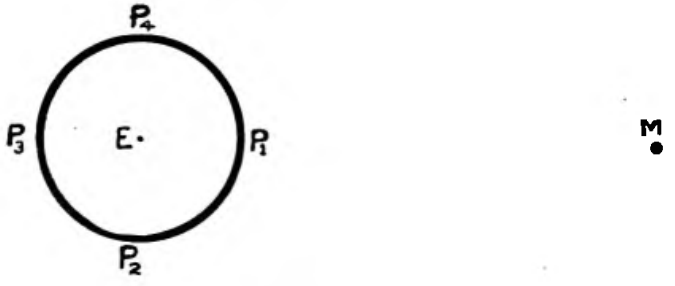


FIG. 5.—LUNAR FORCES TO WHICH A PARTICLE OF WATER IS SUBJECT

earth as a whole. For, according to the law of gravitation, the intensity of attraction decreases with increasing distance. As a result, it follows that there is a force tending to pull the water at P_1 away from the earth, or tending to increase the distance between the earth and the water at P_1.

About 6 hours later the moon will be on the horizon for our point and its position with regard to the moon will then be indicated by P_2. The distance of the moon from the center of the earth is now very nearly equal to its distance from P_2. The moon is therefore pulling the solid earth and the water at point P_2 with the same force and

hence there is now no force tending to pull the water at P_2 away from the earth.

Some 6 hours later still, or a little over 12 hours from the time when our point was directly under the moon, the condition of affairs will be as represented by P_3. Our point is now again directly under the moon, but on the opposite side of the earth from the position P_1. What forces now are brought into play, as regards the solid earth and the water at our point P? Obviously, since the moon is now nearer to the center of the earth than to the water at P_3, its intensity of attraction for the earth is now greater than the intensity of its attraction for the water at P_3. There results, therefore, a force tending to pull the earth away from P_3, or, in other words, tending to increase the distance between the earth and the water at P_3. *But that was exactly what we found to be the case when the moon was directly overhead in position* P_1.

Finally, when the position of our point is P_4, that is, when the moon is rising on the horizon, the distance of the moon from the center of the earth and from P_4 is again practically the same, so that there is now no force tending to pull the water at P_4 away from the earth. And this was likewise the case when our point was in position P_2. We thus find that, although the moon's motion is such that it passes over the meridian of any given place once every 24 hours and 50 minutes, the forces brought about by the differences of the moon's attraction for the solid earth and for the overlying waters are such as to produce two full cycles during that period. Hence, from one meridian passage of the moon to the other, that is, in a period of 24 hours and 50 minutes, we have two high and two low waters.

In explaining the relation between moon and tide, it is frequently considered sufficient to imply, or even to state, that the attractive forces outlined in the preceding para-

graph produce high water under the moon (both on the side nearest the moon and on the side furthest from the moon), while low water is produced when the moon is in the horizon rising or setting. Indeed, the usual explanation might be stated as follows: When the moon is directly over any point of the ocean it attracts that point with greater intensity than it does the solid earth; hence, it pulls the water away from the earth, producing high water. At the same time, however, it is attracting the solid earth with a greater intensity than the water on the opposite side of the earth, and hence pulls the earth away from the water there, thus producing high water there also. But a little consideration will show that this is not a correct explanation.

In the first place, the tides themselves furnish testimony against that explanation, for high water does not occur when the moon is directly overhead. As a matter of fact, high water occurs at all possible times before and after the moon's meridian passage. Taking our own coasts we find high water following the moon's meridian passage by the following intervals: 11½ hours at Boston, 8 hours at New York, 7 hours at Atlantic City, 1½ hours at Philadelphia, 10 hours at Key West, 9½ hours at San Diego, 4½ hours at Seattle. Mention any interval between 0 and 12 hours, and some place can be found where that is the interval by which high water follows the moon's meridian passage.

Another and even more serious objection that can be urged against the usual explanation of the making of the tide is that when the moon is directly overhead its upward pull actually can have no effect in bringing about the tide—in fact at that moment its tide-producing effect on the water directly beneath it is zero. For when the moon is directly above any point it merely reduces the pull of the earth on the water at that point; but at that instant the moon could move the water away from the earth only if its

pull were greater than the pull of the earth. And, as a matter of fact, when the moon is directly overhead it pulls the water beneath it with a force that is only one ten-millionth as great as the pull of the earth on the water.

Suppose we examine more fully the force exerted by the moon on our point P as its position changes in consequence of the rotation of the earth and the revolution of the moon. In position P_1 the moon pulls the water directly upward, or directly opposite to the pull of the earth, and can therefore not move the water. But in all positions between P_1 and P_2, and likewise between P_4 and P_1, it is evident that the pull of the moon is partly vertical and partly horizontal; and, while it requires a very great force to lift the water vertically away from the earth, it is obvious that because of the great mobility of water even a very small force can produce motion horizontally. It is therefore the horizontal component of the moon's attraction that puts the water of the ocean into motion and brings about the tide. From moonrise to moon culmination, that is, from the time our point moves from P_4 to P_1, a period of 6 hours and 12 minutes, this horizontal force is directed to the east and urges the water in that direction. Directly under the moon at P_1, the moon's force is altogether vertical and the horizontal force vanishes.

In the following period of 6 hours and 12 minutes, from moon culmination to moonset—from position P_1 to P_2—it is evident that the horizontal force of the moon's attraction is directed toward the west and urges the water in that direction. We thus have the moon during the period of 12 hours and 25 minutes from moonrise to moonset pulling the water horizontally first to the east and then to the west. Now what is the direction of the horizontal force when the moon is below the earth, when our point is moving from position P_2 to P_3, and from the latter to P_4?

In position P_2 the horizontal force of the moon is nil;

but as our point changes its position from P_2 to P_3 the horizontal force is directed to the east and this can be shown easily by the aid of mathematics; but here we must accept it without proof. At P_3 the attractive force of the moon is again wholly vertical and the horizontal force is zero; but from position P_3 to P_4 the horizontal force of the moon's attraction is directed toward the west.

In the period of 24 hours and 50 minutes, which the moon requires for making its apparent daily revolution around the earth, we therefore have the water at our point P urged alternately to the east and then to the west for periods of 6 hours and 12 minutes. In a lunar day, there are thus two full cycles of these forces, each cycle occupying 12 hours and 25 minutes. And it is these horizontal forces which cause the rhythmic movements of the ocean which we call the tide. For 6 hours and 12 minutes the movement of the water is toward the land and at the end of that period high water occurs; during the following period of 6 hours and 12 minutes the movement of the water is away from the land, causing low water at the end of the period. High water and low water thus follow each other at intervals of 6 hours and 12 minutes, giving two highs and two lows in a day.

In tracing out the forces to which our point P was subject, the point was taken on the earth's equator for the sake of simplicity. For a point not on the earth's equator, a little reflection will show that the horizontal force of the moon will also have a north and a south component. Thus, when the moon is over any given meridian, it is evident that for all points on the earth to the north of the equator there will be a southerly component to the moon's horizontal force, while at the same time all points south of the equator will experience a northerly horizontal force. Six hours later, when the moon is on the horizon, the difference in the attraction of the moon for the solid earth

and for the water gives rise to a northerly component in the northern hemisphere and to a southerly component in the southern hemisphere. The magnitudes of the east and west components as well as of the north and south components vary with the latitude; but it is important to note that at any point on the earth's surface the moon's horizontal tide-producing forces go through their cycle in a period of 12 hours and 25 minutes.

For any point on the earth's surface the magnitude and direction of these tide-producing forces can be computed mathematically from well-established astronomical data. The manner in which these horizontal tide-producing forces vary for different latitudes and the changes they undergo from hour to hour are well shown in Figure 6. Here the line from the center of the curve to each hour gives by its length the magnitude of the force and by its direction the direction in which the force is acting. It is to be remembered that the hours here are lunar hours, 0 hours being the time when the moon is directly overhead. Since the period of these tidal forces is half a day they are called semidiurnal forces.

The tide-producing forces of the sun can be derived in exactly the same way as those of the moon. In fact, by substituting the sun for the moon in Figure 5 and remembering that the sun's apparent revolution around the earth takes place in a period of 24 hours instead of 24 hours and 50 minutes which the moon requires, we will arrive at similar results. The sun's horizontal tide-producing forces will tend to produce two high and two low waters in a period of 24 hours, the interval between the high or low waters being exactly 12 hours.

We are now in a position to understand why the height of the tide varies with the moon's changing phases. At new moon, sun and moon are over the meridian of any given point at the same time, and hence their tide-producing

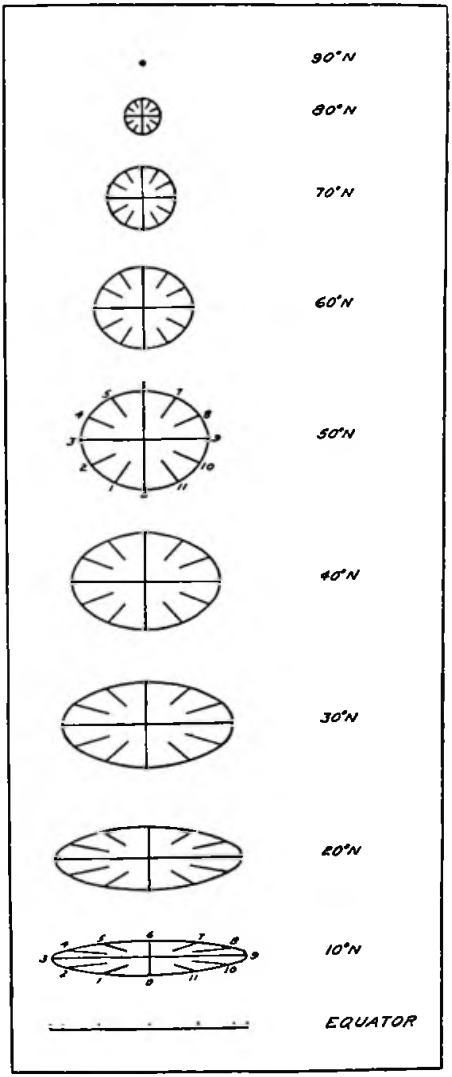


FIG. 6.—THE SEMIDIURNAL TIDE-PRODUCING FORCES (AFTER HARRIS)

forces are acting in the same directions, causing an increased motion of the water. We therefore have greater tides at such times. At full moon, sun and moon are on opposite sides of the earth; but, as we have found, the tide-producing forces are the same when the moon (or sun) is directly above or below any point. Hence, at full moon likewise the tide-producing forces of sun and moon are acting in the same directions and producing increased motion in the waters of the ocean and consequently greater tides.

When the moon is in its first or third quarters, sun and moon are 6 hours apart. Hence, when the sun is urging the waters eastward, the moon is urging them westward, and vice versa. At such times, therefore, the tide-producing forces of sun and moon are acting in opposite directions, the result being a smaller tide-producing force and therefore smaller tides. To take a concrete case, it has been found from observations covering a period of many years, that in New York Harbor the tides are about 20 per cent greater than the average when the moon is full or new, and about 20 per cent smaller than the average when the moon is in its first and third quarters.

According to the law of gravitation the force with which one body attracts another decreases with an increase in the distance between them. The moon revolves around the earth in an ellipse, the period of revolution being about 27½ days. During this period she is at different distances from the earth. When the moon is nearest the earth, or in perigee, her attraction for the waters of the earth should be greater than when she is furthest from the earth, or in apogee. We should therefore expect to find greater tides than the average at perigee and smaller than average tides at apogee. As a matter of fact this is found to be the case. Taking New York Harbor again, it has been found that when the moon is in perigee, that is, when she is nearest

the earth, the tides are about 20 per cent greater than the average; and when the moon is in apogee the tides are about 20 per cent less than the average.

Now it is to be remembered that the month of the moon's phases, or the period from one full moon to another, is 29½ days, while the period from one perigee to another is 27½ days. Hence the relation of full moon to perigee is continually changing, and there will be times when the moon is both full or new and also in perigee. At such times we should therefore expect the tides to have an unusually great rise and fall. Likewise there will be times when the moon is in apogee and also in her first or third quarter; at such times we should expect to find unusually small tides. Suppose we test these theoretical conclusions by actual observations.

In the Narrows, the entrance to New York Harbor, the tides are well determined. At Fort Hamilton, on the eastern shore of the Narrows, an automatic tide gauge has been in operation for over a quarter of a century, furnishing a continuous record of the rise and fall of the tide night and day, winter and summer. The average rise and fall of the tide, that is, the range between mean high and mean low water has been found to be 4.7 feet. At times of new and full moon the rise and fall of the tide is increased by 20 per cent, while at the times of the moon's first and third quarters the range of the tide is decreased by 20 per cent. Furthermore, when the moon is in perigee the tide is 20 per cent greater than the average, while when the moon is in apogee there is a decrease of 20 per cent from the average.

It follows therefore that at any time when the moon is both full or new and in perigee the rise and fall of the tide at Fort Hamilton should be about 40 per cent greater than the average, while at times when the moon is in apogee and also in her first or third quarter the tide should be

40 per cent less than usual. Looking through a nautical almanac for the year 1916 we find the moon to have been full and also in perigee on July 15. And in the following period of 24 hours the tide observations at Fort Hamilton show that the rise and fall of the tide was 6.5 feet or 39 per cent greater than the average range of 4.7 feet. In 1916, too, the moon was in apogee and also in her third quarter on October 19. And on that day the tide rose and fell through a vertical distance of only 2.8 feet or 40 per cent less than the average of 4.7 feet.

The tide-producing forces of sun and moon are a consequence of the gravitational attraction which Newton formulated in the so-called law of gravitation. This law states that two bodies attract each other with a force which varies directly as their masses and inversely as the square of the distance between them. We find, however, that while responding to the sun's tide-producing forces the tide nevertheless appears to be governed very largely by the moon.

Now this is something of a paradox; for compared with the sun the moon is, in size, quite insignificant. The sun can furnish material to make 26,000,000 moons and still have enough left to make something like 10,000 planets of the size of our earth. Why is it then that the moon plays the leading rôle in the production of the tides, making the length of the tidal day the same as the lunar day and compelling the tide to keep step with her own movement? What explains this seeming paradox?

The explanation is found in the fact that the tide-producing forces are due to *differences* in the attraction of sun and moon for the earth as a whole and for the water covering the earth. When the mathematician works out the law that governs the tide-producing power of a heavenly body, he finds that there are two parts to it. First, the power to produce tides varies exactly as the amount of

matter in the body, the greater the body the greater the tidal force. Second, the further the body the less the tidal force, but in such manner that when the distance is doubled the tidal force is reduced eightfold; or, as the mathematician puts it, the tide-producing power of a body varies directly as its mass and inversely as the cube of its distance.

Applying these laws of the tides to the sun and moon, we find that because of its enormous mass the sun's tide-producing power should be more than 26,000,000 times as great as that of the moon, if they were both at the same distance from the earth. But the astronomer tells us that as regards distance from the earth, the moon is 389 times nearer than the sun. Its advantage over the sun because of its nearness is found by multiplying 389 by itself twice, that is, "cubing" 389, which gives 58,863,869. The moon, therefore, because of its nearness overcomes the advantage of the sun's greater mass in the proportion of 58,000,000 to 26,000,000; that is, the moon's force to produce tides is something like 2¼ times as great as that of the sun, and hence the tide follows the moon.

The question may be raised whether the tides are not influenced by other heavenly bodies. The laws of the tide mentioned above permit us to determine this. The nearest planet to the earth is Venus, which, compared with the moon, has a mass about 70 times as great, but is 100 times further from the earth. Hence, the tide-producing force of Venus as compared with that of the moon is given by the fraction $\frac{70}{100\times100\times100}$. In other words the tide-producing force of Venus is less than one ten-thousandth that of the moon. If the moon brought about tides having a range of 100 feet, we might expect a Venusian tide of something like one-tenth of an inch. Actually, therefore, the tidal effect of Venus on our earth is practically nil.

If we consider the planet having the greatest mass,

Jupiter, we find that when it is nearest the earth its distance is about 370,000,000 miles or in round numbers about 1,500 times as far away as the moon. Its mass is 26,000 times that of the moon; hence, its tide-producing force on the waters of the earth as compared with that of the moon is given by the fraction $\frac{26,000}{1,500 \times 1,500 \times 1,500}$. This gives for Jupiter a tide-producing force less than one one-hundred-thousandth that of the moon. Of all the heavenly bodies, therefore, the sun and moon are the only ones concerned in bringing about the rise and fall of the tides on our earth.

We have found that the vertical component of the moon's attractive force is not concerned in the making of the tide; but its effect is nevertheless felt by all bodies on the earth. This vertical force is greatest at a point directly under the moon, the moon's attraction there being wholly upward. Its effect, therefore, is to diminish the downward pull of the earth or, in other words, to diminish the weight of a body at that point. It has been pointed out previously that the upward pull of the moon when she is directly over any point of the earth's surface is about one ten-millionth part of the downward pull of the earth on that same point. Hence, when the moon is shining vertically down on any body it is taking away one ten-millionth of that body's weight. A steamer of the weight of the *Majestic*—56,000 tons—therefore loses something like 15 pounds of its weight when the moon is directly above it.

Thus far in our consideration of the tide-producing forces we have not taken into account the fact that the moon moves around the earth in an orbit which is inclined to the plane of the equator. Instead of being always over the equator, the moon is sometimes north of it and sometimes south. This introduces a factor into our tide-producing forces which we must now proceed to consider.

In Figure 6 the variations of the semidiurnal tide-pro-

ducing forces were shown for various latitudes at each lunar hour. Suppose, however, we desire to picture to ourselves the distribution of the tide-producing forces over the entire earth at any given instant. For such a purpose, the graphic representation of Figure 7 gives a clearer view of the matter. Here the circle represents half the earth from pole to pole, the length of the arrows indicating

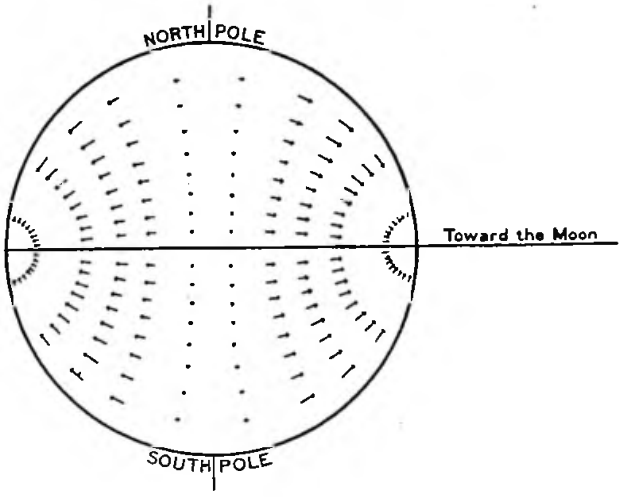


FIG. 7.—DISTRIBUTION OF TIDE-PRODUCING FORCES, MOON ON EQUATOR (AFTER HARRIS)

the intensity of the tide-producing force and the direction of the arrows the directions in which the force is acting.

We have found that the tide-producing forces, with respect to the position of the moon, are identical for points on the earth directly under each other. We may therefore conceive the tide-producing forces as brought about by two moons situated on opposite sides of the earth. To make this assumption consistent with the facts we must further

assume that every point on the earth's surface is subject to the tidal forces of one of these moons for 12 lunar hours and to the tidal forces of the other moon for the succeeding period of 12 hours. For convenience, the hypothetical moon on the side of the earth opposite the real moon is called the "anti-moon." In Figure 7 the position of the real moon is indicated to the right of the figure and the position of the anti-moon to the left.

As the earth rotates on its axis it is obvious that when the moon is on the equator the successive positions of any given point on the earth will, with respect to moon and anti-moon, be identical for the 12 hours that it is subject, in turn, to the tidal forces of moon and anti-moon. In other words, when the moon is on the equator the tide-producing forces repeat themselves exactly every 12 lunar hours, and we should therefore expect morning and afternoon tides to be very much alike at such times. This is actually the case. But what is the condition of affairs when the moon is not on the equator?

Figure 8 represents the distribution of the tide-producing forces when the moon is north of the equator. Relative to moon and anti-moon the distribution of the forces is identical with that shown in Figure 7, when the moon is on the equator. But the rotation of the earth now makes the successive positions of any point on the earth quite different for the two 12-hour periods during which it is subject, in turn, to the tidal forces of moon and anti-moon. For the path traced out by a point on the earth's surface during the rotation of the earth is a circle parallel to the plane of the equator which now is inclined to the plane of the moon's orbit.

In consequence of the inclination of the moon's orbit, the tidal forces do not exactly repeat themselves at intervals of 12 hours when the moon is north or south of the equator. It is only when the earth has completed a full

revolution that the tide-producing forces for any point are again the same. The tidal forces due to the moon are therefore of two kinds, the first having a period of half a lunar day and the second having a period of a whole lunar day. The former are known as semidiurnal forces and the latter as diurnal forces. The diurnal forces for any point on the earth's surface, like the semidiurnal forces, can be computed from well-established astronomical

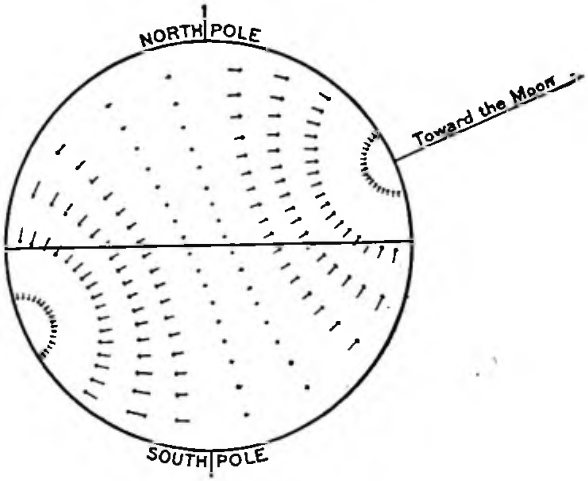


FIG. 8.—DISTRIBUTION OF TIDE-PRODUCING FORCES, MOON NORTH OF THE EQUATOR (AFTER HARRIS)

data. The manner in which these diurnal tide-producing forces vary for different latitudes when the moon has its greatest north or south declination and the changes they undergo from hour to hour are shown in Figure 9. The line from the center of the curve to each hour indicates by its length the magnitude of the force and by its direction the direction in which the force is acting. It is to be remembered that the hours here shown are lunar hours, 0

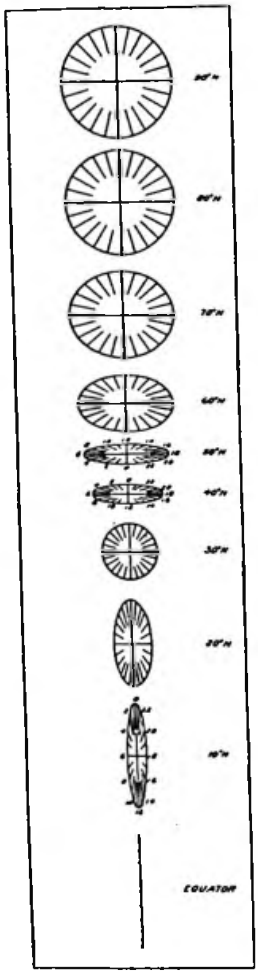


FIG. 9.—THE DIURNAL TIDE-PRODUCING FORCES (AFTER HARRIS)

hours being the time when the moon is over the local meridian. It should be noted, too, that from the equator to 45° north latitude the direction of the tide-producing forces changes counterclockwise, while north of 45° the change is clockwise.

Since the tidal forces repeat themselves every 12 hours when the moon in on the equator, we should expect, at such times, morning and afternoon tides to be alike. When the moon is not over the equator, the tidal forces of the first 12 hours are not quite the same as those of the following 12 hours, and hence at such times the morning and afternoon tides should be somewhat different. These conclusions are borne out by tidal observations, though it is to be noted that not the same extent at all places. For example, the differences between morning and afternoon tides, when the moon is north or south of the equator, are far greater on the Pacific coast than on the Atlantic. Figure 10 gives in diagrammatic form the tide curves for San Francisco on the 18th and 24th of October, 1922. On the former date the moon was on the equator, and on the latter date the moon had its greatest south declination for the month.

The orbit of the sun like that of the moon is inclined to the plane of the equator. Of the sun's tide-producing force, therefore, part is semidiurnal and part diurnal. It is to be remembered, however, that the period of the moon's diurnal force is 24 hours 50 minutes, while the period of the sun's diurnal force is exactly 24 hours. Furthermore,

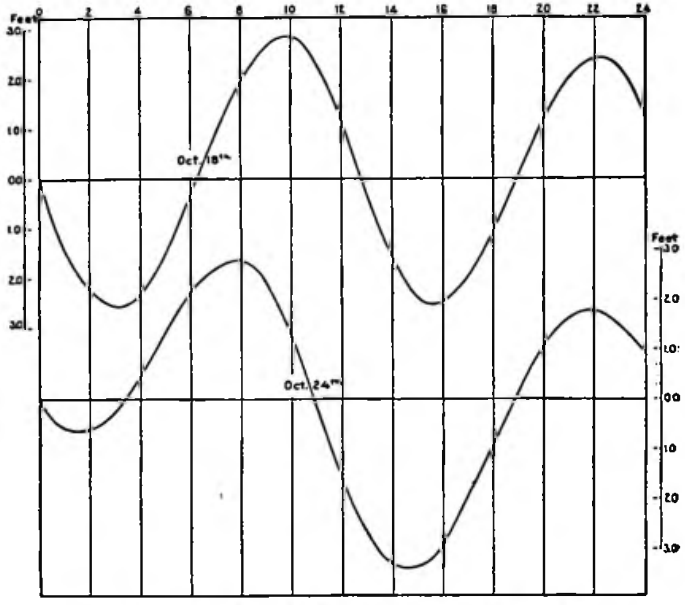


FIG. 10.—TIDE CURVES, SAN FRANCISCO, CALIFORNIA, OCTOBER 18TH AND 24TH, 1922

the tide-producing power of the sun is considerably less than that of the moon; so that while the diurnal forces of the sun tend to produce an inequality in the two tides of the day, when the sun is not on the equator, the moon's diurnal forces are the predominating constituents.

The tides are brought about by the tide-producing forces

of sun and moon which we have been considering in this chapter, and we have found that these tide-producing forces vary with variations in the relative positions of sun, moon, and earth, and that in response to these changes in the tide-producing forces changes in the rise and fall of the tide take place. It will now be of advantage to consider in greater detail the response of the sea to the tide-producing forces of sun and moon—to study the tides as they actually occur in nature.

CHAPTER IV

THE RESPONSE TO THE TIDE-PRODUCING FORCES

If the earth were entirely covered with a layer of water of considerable depth, the mathematician assures us that the tides would present phenomena of a regular and uniform character. Everywhere high water would occur about the time of the moon's local meridian passage and low water 6 hours later; the extent of rise and fall of the tide would vary strictly with the latitude, being greatest near the equator and least at the poles, and, furthermore, the tide at all places having the same latitude would have absolutely similar characteristics. In other words, in this ideal world the time and characteristics of the tide at any point would be completely specified as soon as its latitude and longitude were known.

But the tides as they actually present themselves to us in nature have nothing of the simplicity of the tides of the mathematician's ideal world entirely covered with water. High water and low water occur at all possible hours before and after the moon's meridian passage and with complete disregard of longitude. Likewise the extent of rise and fall of the tide is found to bear no relation whatever to the latitude; in fact, places having the same latitude frequently have tides differing very considerably, while places having different latitudes may have tides very much alike. Thus, St. John's, Newfoundland, and Brest, France, face each other across the Atlantic Ocean on very nearly the same parallel of latitude. At St. John's high water comes 7 hours after the moon passes overhead, and the rise

and fall varies from 1½ to 3½ feet, while at Brest high water comes 3½ hours after the moon's meridian passage, and the extent of rise and fall is nearly 15 feet on the average and frequently exceeds 18 feet.

On the other hand, if we compare the tides at St. John's with those at Matanzas Inlet on the Atlantic coast of Florida we find them very much alike although separated by 18° or more than 1,200 miles of latitude. In extent of rise and fall, in relation to the time of the moon's meridian passage and in general characteristics the tides at the two places resemble each other closely.

If we compare the tides on the Atlantic and Pacific coasts of the United States, we find differences of a still more profound character. Eastport on the coast of Maine and Cape Foulweather on the coast of Oregon are very nearly on the same parallel of latitude. The tide at Eastport has a rise and fall of 18 feet or more and is of the type in which morning and afternoon tides are very much alike, while at Cape Foulweather the rise and fall of the tide is but 6 feet and is of the type in which morning and afternoon tides show considerable differences.

An unusually good example of the complete disregard of latitude and longitude is furnished by the tides at the two ends of the Panama Canal. Though little more than 30 miles apart as the crow flies, the Pacific and Atlantic ends of the Canal have totally different tides. At Colon, on the Atlantic side, the tides have a rise and fall of less than a foot on the average, and generally there is but one high and one low water in a day. At Balboa on the Pacific side of the Canal, two high and two low waters occur during the day and the tide has a rise and fall of from 12 to 16 feet.

The distribution of the tidal forces on the earth we have found to be in strict accordance with latitude; but a glance at the tides as they actually occur proves conclusively that the response of the sea to these tide-producing forces is

not of as simple and regular a character. What is to be borne in mind is that the continents and islands rising out of the oceans divide the waters of the earth into a number of irregular-shaped basins of varying depths. It is therefore not surprising that the tides at different places should be unlike; for is it not reasonable to expect the rise and fall of the water on the shores of a broad and deep ocean basin to be different from the tides in a shallow restricted basin, although in both cases brought about by the same tidal forces of sun and moon?

The fact to be emphasized is that the tidal *forces* are of astronomic origin and may be computed from a general formula, since they depend entirely upon the relative masses and positions of sun, moon, and earth; but the *tides* brought about by these forces are so very profoundly modified by terrestrial features that the characteristics of the rise and fall of the water at any given point are determined by local regional features. No general formula can therefore be derived by means of which the time and height of the tide at any given point on the earth may be computed. Looking at the matter in another way, we may say that the study of the tide-producing forces falls naturally under the subject of astronomy, while the study of the tides as they actually occur comes under the province of geography.

Manifestly, we may begin the study of the response of the various seas to the tide-producing forces of sun and moon at any desired place, but there are many reasons for studying the tides on our own shores first. Let us therefore begin with the rise and fall of the sea on the Atlantic coast of the United States. From Maine to Florida the winding coast line with its numerous indentations exposes to the tide of the Atlantic a shore line some 5,000 miles in extent. Along this extensive shore line the characteristics of the tide are quite well determined. Tidal observations have been made at numerous points; and at certain selected

places automatic tide gauges have been in operation continuously for a number of years.

The ocean tide as it enters the various harbors on the Atlantic coast is modified by the configuration and hydrographic features of the harbors to a very considerable extent. If we desire to study the tide in its unaffected condition—in its natural state, so to speak—we must secure observations on the open coast. But this is not a simple matter; for the securing of tidal observations generally requires some sort of wharf structure on which to place the automatic tide gauge, and wharves are very rarely found on the open coast. Fortunately, however, there are on the coast of New Jersey a number of amusement piers jutting out several hundred feet into the open ocean. At Atlantic City a tide gauge has been in operation on one of these piers for a number of years, and the record thus secured furnishes us with data for the beginning of our study of the Atlantic coast tides.

Since the length of the tidal day is the same as that of the lunar day, namely, 24 hours 50 minutes, it is convenient to refer the rise and fall of the tide to the apparent daily revolution of the moon. From observations covering a period of 9 consecutive years, it has been found that at Atlantic City, on the average, low water comes exactly 1 hour after the moon crosses the local meridian, and high water 7 hours 13 minutes after the moon's meridian passage. The average rise and fall or mean range of the tide is 4.0 feet, but the morning and afternoon tides are generally somewhat different. Figure 11 represents in graphic form the average rise and fall of the tide with reference to the moon's meridian passage, 0 hours being the instant when the moon is directly over the meridian of Atlantic City.

It is to be noted that the two high waters of the day do not rise to the same height above sea level and that the two

low waters do not fall the same distance below sea level. This variation in the heights is known as "diurnal inequality," and, as previously pointed out, is due to the fact that the orbits of sun and moon are inclined to the plane of the equator. In virtue of the moon's greater tidal force, the diurnal inequality varies throughout the month very largely as the declination of the moon. When she is near the equator the morning and afternoon tides are very much alike, the diurnal inequality becoming very small; when she has reached her maximum semimonthly north or south

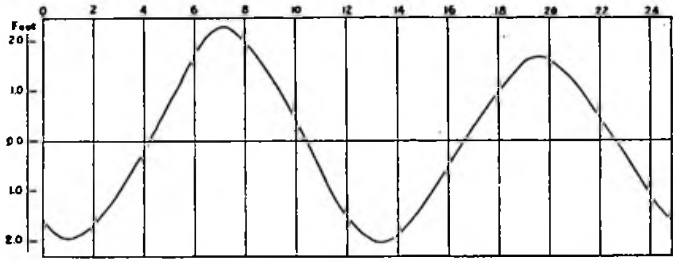


FIG. 11.—TIDE CURVE REPRESENTING AVERAGE RISE AND FALL OF THE TIDE AT ATLANTIC CITY, NEW JERSEY

declination, the diurnal inequality attains its greatest value, and the tides are then called "tropic tides" since the moon is then near one or the other of the tropics.

On the average the two high waters of any one day at Atlantic City differ by 0.7 foot, and the two low waters by a little less than 0.1 foot. When the moon is at its semimonthly maximum north or south declination the difference in the heights of the two high waters of the day becomes 1.2 feet, while in the low waters it becomes a little over 0.1 foot. At Atlantic City, therefore, it is the high water that features the diurnal inequality in the height of the tide. But it is important to note that even in the case of the

high waters this inequality is relatively small as compared with the range of the tide.

At times of full and new moon, when the tidal forces of the sun and moon are conspiring and the so-called "spring tides" occur, the rise and fall of the tide at Atlantic City is increased about 20 per cent, the range then becoming 4.8 feet, and when the moon is in its first and third quarters, when "neap tides" occur, the rise and fall of the tide is decreased about 20 per cent, the range then being 3.1 feet.

The rise and fall of the tide, as we have found, responds also to the moon's varying distance from the earth. When in its monthly revolution around the earth the moon is nearest the earth, or in perigee, the range of the tide at Atlantic City is increased 22 per cent, becoming then 4.9 feet. When the moon is furthest from the earth, or in apogee, the range is decreased 18 per cent, becoming 3.3 feet.

From an analysis of the tidal observations at Atlantic City it has been found that the tide does not respond immediately to variations in the position of the moon relative to earth and sun. The greatest rise and fall of the tide does not take place on the day when the moon is full or new but a day later, and likewise neap tides occur a day after the moon's first or third quarters. Again, it is about 1½ days after the moon is in perigee that the tide has its greatest range and 1½ days after the moon's apogee that the range is least. This retard in the response of the tide is called the "age of the tide" and is generally ascribed to the effects of friction.

While, on the average, low water at Atlantic City comes one hour after the moon's transit and high water 7 hours 13 minutes after transit, these intervals vary somewhat from day to day. There are several causes for this variation, but at this time we need consider but one, namely,

the difference in the periods of apparent revolution of sun and moon about the earth.

The period of apparent revolution of the sun about the earth is 24 hours. And as we found in our consideration of the tidal forces of the sun these tidal forces repeat themselves exactly every 12 hours. Hence, the high and low waters of the tide produced by the sun occur every day at the same hours. But the tides brought about by the moon, in consequence of the moon's period of revolution being 24 hours 50 minutes, tend to become later each day by 50 minutes. It follows therefore that the sun's tide is sometimes earlier and sometimes later than that of the moon. When it is earlier its effect is to make the combined tide somewhat earlier than the average, and hence the interval by which the tide follows the moon at such times is less than the average; when the sun's tide is later than that of the moon, the interval by which low water or high water follows the moon's transit is greater than the average. But it is to be noted that at Atlantic City it is only infrequently that the interval varies by as much as 60 minutes from its average value of 1 hour for low water and 7 hours 13 minutes for high water.

We have found (page 35) that the tide-producing force of the moon is about 2¼ times as great as that of the sun. If the Atlantic Ocean, or that part of it from which the Atlantic City tide is derived, responded to the tide-producing forces of sun and moon in the same proportion we should expect the tide raised by the moon to be 2¼ times as great as that brought about by the sun. How can we separate the two tides at Atlantic City? Very easily, from a consideration of the spring and neap tides. Spring tides occur very soon after the moon is full or new, that is, when the tidal forces of sun and moon are acting in unison, and the range of the combined tide at such times is, from observation, 4.8 feet. Neap tides occur soon after the moon

is in its first or third quarters, when the tidal forces of sun and moon are opposed, and at these times the observed tide has a range of 3.1 feet. If we designate the moon's tide by M and that of the sun by S we have M+S=4.8 feet and M−S=3.1 feet. It is now a matter of simple calculation to determine that M=3.95 feet and S=0.85 feet.

Now 3.95 is just about 4½ times 0.85, that is, the moon's tide at Atlantic City is 4½ times as great as the sun's tide. But the moon's tidal *force* is only 2¼ times as great as that of the sun. We are therefore led to conclude that the Atlantic Ocean, or at least that part of it from which the tide at Atlantic City is derived, responds better to the tidal forces of the moon than to those of the sun.

One other feature of the tide at Atlantic City deserves mention, and that is the duration of rise and of fall. The average duration of rise is 6 hours 13 minutes and the average duration of fall is 6 hours 12 minutes. Practically, therefore, we may say that the two periods are equal. The tide curve for Atlantic City shown in Figure 11 brings this out.

Now if we study the tide along the Atlantic coast of the United States from Maine to Florida we find that the range of the tide varies from place to place and likewise the intervals by which high water and low water follow the moon's meridian passage. To cite but a few instances: At Eastport, Maine, the tide has a range of 18.2 feet, high water coming $11^h\ 09^m$ after the moon's meridian passage; on the southeastern shore of Nantucket Island, Massachusetts, the corresponding figures are 1.2 feet and $10^h\ 23^m$; at the entrance to New York Harbor 4.7 feet and $7^h\ 40^m$; at the entrance to Savannah River, Georgia, 6.8 feet and $7^h\ 11^m$; and at Miami, Florida, 1.4 feet and $9^h\ 30^m$.

However, apart from the range of the tide and the intervals by which high and low water follow the moon's

meridian passage, the Atlantic coast tides from northern Maine to southern Florida exhibit the characteristics enumerated above for the tide at Atlantic City. We may therefore summarize the characteristics of the Atlantic Ocean tide along the open coast of the United States as follows:

1. Two high and two low waters occur during each tidal day ($24^h\ 50^m$), morning and afternoon tides being very much alike.

2. The diurnal inequality in the height of the tide exists almost wholly in the high waters, the low waters exhibiting very little inequality. But in comparison with the range of the tide the inequality even in the high waters is relatively small.

3. The interval between the moon's meridian passage and high or low water is very nearly constant, only infrequently varying by as much as an hour from the average value.

4. The rise and fall of the tide varies from day to day in accordance with the moon's phases and varying distance from the earth; spring tides and perigean tides are 20 per cent greater than the average, neap tides and apogean tides 20 per cent less than the average.

5. Spring tides and neap tides follow full or new moon and first or third quarters by one day; perigean and apogean tides follow the moon's perigee and apogee by 1½ days.

6. The duration of rise and the duration of fall are equal, each being about 6 hours 12 minutes or 6.2 hours.

The features of the tide enumerated above may be regarded as characterizing the response of the North Atlantic Ocean to the tide-producing forces of sun and moon. Let us now investigate the characteristics of the response of the Pacific Ocean to these same tidal forces, as they are exemplified by the tide on the Pacific coast of the United States.

Although the North Pacific Ocean washes more than a thousand miles of our coast line from the Mexican border on the south to the Canadian boundary on the north, we have, unfortunately, no long-continued record of the tide on the open coast. But there is an excellent series of tidal observations in the Golden Gate, the entrance to San Francisco Bay, and these observations may be used for our study of the tides on the Pacific coast.

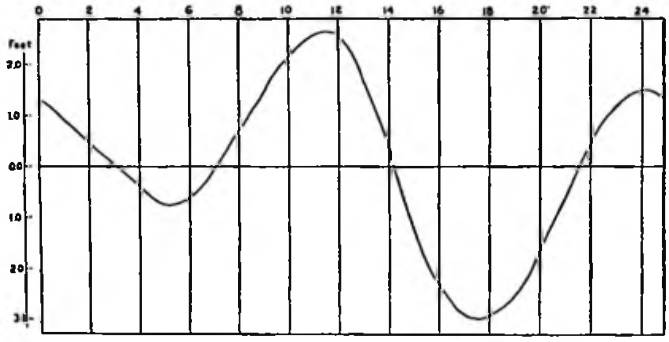


FIG. 12.—MEAN TIDE CURVE, GOLDEN GATE, SAN FRANCISCO HARBOR

In Figure 12 there is represented the average rise and fall of the tide in the Golden Gate. Low water comes $5^h\ 03^m$ after the moon crosses the local meridian and high water $11^h\ 39^m$ after the moon's meridian passage, the mean range of the tide being 3.9 feet. At first glance this tide curve looks much like that for Atlantic City on page 47, but a closer examination reveals the fact that although the difference between the heights of the two high waters at both places is very nearly the same—0.7 feet at Atlantic City, 0.6 feet in Golden Gate—the difference in the heights of the two low waters of the day is decidedly different. At Atlantic City we found this

difference (Fig. 11) to be 0.1 foot; in the Golden Gate Figure 12 shows this difference to be 1.2 feet.

The diurnal inequality varies throughout the month in accordance with the moon's changing declination. Twice each month therefore—when the moon reaches its maximum north and also when it reaches its maximum south declination—the diurnal inequality is greatest, and at such times the tides are known as tropic tides. At Atlantic City we found that the tropic tides show an average difference of 1.2 feet in the two high waters of the day and 0.1 foot in the two low waters. Expressing these height differences as percentages of the mean range of the tide we may say that the diurnal inequality in the height of the tropic tide at Atlantic City is, for the high waters, 30 per cent of the range and for the low waters 2 per cent.

Now in the Golden Gate we see that the *mean* tide has a greater inequality than the tropic tide at Atlantic City. We should, therefore, expect the tropic tide to show even more pronounced inequality. And this is borne out by observations. Figure 13 represents the average conditions of the rise and fall of the tide in the Golden Gate at the times of the moon's maximum semimonthly north or south declination.

For the tropic tide in the Golden Gate Figure 13 shows that the high waters have an average height difference of 1.3 feet and the low waters a difference of 3.4 feet. Expressing these differences as percentages of the mean range of the tide we find the diurnal inequality to be 33 per cent of the range of the tide for the high waters and 87 per cent for the low waters.

The diurnal inequality in the heights is thus a much more pronounced feature of the tide in the Golden Gate and in this respect differs markedly from the tide on the Atlantic coast. Furthermore, on the Atlantic coast the diurnal inequality exists almost wholly in the high waters,

while in the Golden Gate the inequality is three times as great in the low waters as it is in the high waters.

Where the tide exhibits such marked differences between consecutive high waters or low waters, it is obviously convenient to have distinctive names for each of these tides. Of the two high waters of any day, the higher one is called the "higher high water," and the lower one the "lower high water." Similarly, of the two low waters of any day the lower one is called the "lower low water" and the

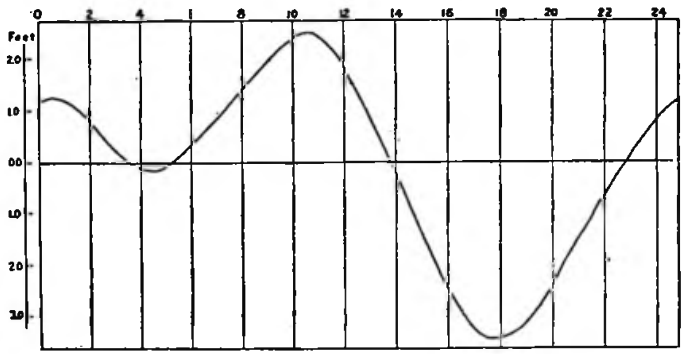


FIG. 13.—TROPIC TIDE CURVE, GOLDEN GATE, SAN FRANCISCO HARBOR

higher one the "higher low water." For the tropic tide in the Golden Gate it is interesting to note that, on the average, the higher low water is about mean sea level.

In another respect do the tides in the Golden Gate differ from the tides on the Atlantic coast, and that is in their time relations. On the Atlantic coast of the United States we found that the high waters, and likewise the low waters, follow the moon's meridian passage by very nearly constant intervals—only infrequently varying by as much as an hour from the average value. This means that on the Atlantic coast the interval between two high or two low

waters is approximately $12^h\ 25^m$, and only infrequently does this interval differ by more than one hour from that value. In the Golden Gate, however, Figure 13 shows that for the tropic tides the intervals between consecutive high or low waters differ considerably from the average value of $12^h\ 25^m$. From lower high water to higher high water the interval is 10 hours, and from higher high water to lower high water the interval is $14^h\ 50^m$. Likewise, there is an interval of $13^h\ 30^m$ from higher low water to lower low water and $11^h\ 20^m$ from lower low water to higher low water.

As we shall see later, when considering the immediate causes of the wide differences of diurnal inequality in the tides of the various oceans, diurnal inequality in the height of the tide must be accompanied by diurnal inequality in the time of the tide. Furthermore, it may be taken as a general rule that the greater the inequality in the height of the tide, the greater the inequality also in the time of the tide. On the Atlantic coast the diurnal inequality in the height of the tide is small; hence the diurnal inequality in the time of the tide is small.

In other respects the tide in the Golden Gate is very much like the tide on the Atlantic coast. Spring tides are 20 per cent greater than the average and neap tides 20 per cent less than the average. Similarly, perigean tides are 17 per cent greater than the average and apogean tides 15 per cent less than the average. Spring tides in the Golden Gate come about 5 hours after the moon is full or new, whereas on the Atlantic coast this interval is about one day. Perigean tides follow the moon's perigee by an interval of 2 days at San Francisco, thus being about half a day later than on the Atlantic coast. In regard to the periods of rise and fall at San Francisco, the duration of rise is somewhat greater than that of fall, but both are approximately 6 hours.

It is, therefore, only in three features that the tide in the Golden Gate differs markedly from the tide on the Atlantic coast of the United States. At San Francisco the tide exhibits considerable diurnal inequality in the time of tide, whereas on the Atlantic coast there is very little; the diurnal inequality in the height of the tide at San Francisco is considerably greater than on the Atlantic coast; while there is diurnal inequality in both the high and low waters at San Francisco, the low water inequality is the greater, whereas on the Atlantic coast, the high water inequality is the greater.

The tide in the Golden Gate may be taken as typifying the tide on the Pacific coast of the United States. From San Diego on the south to the Strait of Juan de Fuca on the north, the tide on the Pacific coast varies considerably in time, in range, and in magnitude of diurnal inequality; but throughout this stretch the tide exhibits considerable diurnal inequality in time of tide, marked diurnal inequality in height of both high and low waters and the greater inequality in the heights of the low waters. On the Pacific coast of the United States, therefore, the response to the tide-producing forces of sun and moon is, in the main, much like that on the Atlantic coast; nevertheless striking differences are exhibited in certain features.

There is still another tidal body of water that washes the shores of the United States. On the southeast, the Gulf of Mexico forms the boundary. This body of water has an area of very nearly three-quarters of a million square miles, large enough to respond in an appreciable manner to the tide-producing forces of sun and moon. We have found that the tide on the Pacific coast of the United States differs in some respects from the tide on the Atlantic coast. Let us now study the tide in the Gulf of Mexico and determine the character of the response to the tide-producing forces of sun and moon in this body of water.

At the very outset it is important to note that the Gulf of Mexico connects with the Atlantic Ocean, so that the tide from the latter can enter and modify the tide of the Gulf proper. Obviously, the effects of the Atlantic Ocean tide are greatest where the Gulf debouches into the ocean. We shall, therefore, take for our study the tides at a port some distance from the Atlantic. Galveston, Texas, is well

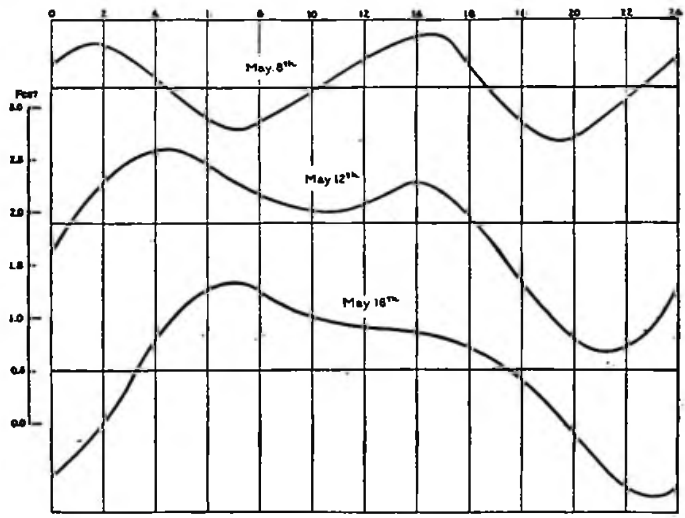


FIG. 14—TIDE CURVES, GALVESTON, TEXAS, FOR MAY 8TH, 12TH AND 16TH, 1915

situated for this purpose, and there is the further advantage that at this port an automatic tide gauge has been in operation for a number of years.

In Figure 14 we have pictured before us the rise and fall of the tide at Galveston, Texas, on May 8, 12, and 16, 1915. The upper curve, for May 8, looks much like the mean tide curve for Atlantic City, shown in Figure 11. Two high and two low waters occur on that day with but little

difference between morning and afternoon tides. The middle curve, for May 16, bears a close resemblance to the curve of the tide in the Golden Gate, shown on page 54. There are two high and two low waters during the day, but morning and afternoon tides show marked differences. The tide curve for May 16, however, is altogether different from anything we have yet considered. Instead of two high and two low waters on that day there is but one high and one low water.

The tide curves show how this has come about. On the 8th of May, there is very little diurnal inequality in the tides, morning and afternoon tides being very much alike. On the succeeding days, the tides begin to exhibit diurnal inequality in greater and greater degree. As shown on the tide curve for the 12th of May, the lower high water and the higher low water differ only three-tenths of a foot in height. The increase in the diurnal inequality continues until on the 16th of May, the higher low water and lower high water no longer differ and therefore merge into each other, so that but one high and one low water occur during the day.

Throughout the month the tide at Galveston goes through the cycle shown in Figure 14. About 25 per cent of the time the tide rises and falls as represented by the upper tide curve. For about 50 per cent of the time the rise and fall of the tide takes place as shown by the middle curve of Figure 14, and the remaining 25 per cent of the time the rise and fall takes place as represented by the lower curve. It is to be noted, however, that the change from one type to the other is not haphazard, but varies in accordance with the position of the moon relative to the equator. When the moon is close to the equator the tide rises and falls as in the upper curve. When the moon is furthest north or south from the equator the rise and fall takes place as represented by the lower curve, one high

and one low water occurring in a day; the remainder of the time, when the moon is neither near the equator nor near the tropics, the rise and fall follows the middle curve.

In other respects the tide at Galveston behaves much like the tide on the Pacific or Atlantic coasts of the United States. The extent of rise and fall of the tide is small, being on the average about one foot, but when the moon is full or new the range of the tide is increased about 20 per cent, and when the moon is in its first or last quarter there is a decrease of about 20 per cent in the range. Likewise, when the moon is nearest the earth there result tides greater than the average, and when the moon is furthest from the earth the tides are less than the average.

Over the greater part of the Gulf of Mexico the rise and fall of the tide exhibits the features found in the tide at Galveston. At some places the percentage of the time during which but one high and one low water occur in a day may be greater than at Galveston and at others this percentage may be less. In general, however, it may be said that the phenomena of the tides at Galveston are the characteristic phenomena of the tides in the Gulf of Mexico.

If now we study the tides as they rise and fall on the shores of the seven seas, we would find that in their principal features they divide themselves into three kinds, typified respectively by the tides on the Atlantic, Gulf, and Pacific coasts of the United States. In other words, the response of the ocean waters to the tide-producing forces is, in the main, largely the same; in certain features, however, striking differences appear, giving rise to different types of tide. In the next chapter we shall investigate these types of tide somewhat more fully and attempt to find some guiding principle to help explain these variations in the response to the tide-producing forces of sun and moon. However, before leaving the subject we have been discuss-

ing, it is necessary to add that thus far we have been considering only the vertical rise and fall of the water, which is but one feature of the response to the tide-producing forces. The other feature, the horizontal movement, it will be more convenient to take up in a later chapter.

CHAPTER V

TYPES OF TIDE

THE tide is the response of the sea to the tide-producing forces of sun and moon. These tide-producing forces, while varying from day to day in accordance with the changing positions of moon and sun relative to the earth, are distributed in a regular manner over the surface of the earth, varying with latitude. The tides, too, vary from place to place, but the differences in the tide appear to bear no relation to latitude, being largely local or regional. What explains this varied response to the same set of tidal forces?

It is important to note that we have not as yet considered the precise nature of the movement of the sea as a whole—the making of the ocean tide—in response to the tide-producing forces. This must be left for a later chapter. What we are at present concerned with is the response to the tide-producing forces as exhibited along the shores of various bodies of water. The tide along the Atlantic coast we have found to be of one type; along the Pacific coast we found a different type and in the Gulf of Mexico a still different type. Where shall we look for a guiding principle to help clarify this seemingly willful response of the various bodies of water to the tide-producing forces of sun and moon?

If we go back to our consideration of the tide-producing forces outlined in Chapter III, we will recall that these forces are of two kinds, semidaily and daily, or in their more technical designation, semidiurnal and diurnal. The

semidaily forces go through a complete cycle in half a day, while the daily forces require a full day to complete their cycle. The tides brought about by these two classes of tide-producing forces are therefore different, the semidaily forces giving rise to a tide having a period of half a day, while the tides brought about by the daily forces have a period of a day. And it is in the varied combinations of the daily and semidaily tides that we find the explanation for the different types of tide.

Let us consider this matter in greater detail. Let us take a point on the coast where the range of the tide is 2 feet and examine the character of the rise and fall of the tide, first on the assumption of wholly semidaily tide-producing forces, then on the assumption of wholly daily forces, and lastly on the assumption of combined daily and semidaily forces.

If the tide-producing forces were wholly of the semidaily variety, the rise and fall of the tide for a tidal day of 24 lunar hours would be represented by the dotted curve of Figure 15. Since the period of the semidaily forces is half a day, there would be two complete cycles in a day and therefore two high and two low waters, morning and afternoon tides being alike in all respects, the high waters rising one foot above and the low waters falling one foot below mean sea level. On the assumption of wholly daily tide-producing forces, the rise and fall of the tide would be represented by the dashed curve of Figure 15, only one high and one low water occurring during the day.

But suppose the tide-producing forces to be partly of the daily and partly of the semidaily variety and let us further suppose that each kind separately would produce a tide with a two-foot range. How would the rise and fall of the tide appear now? Obviously, the height of the tide at any moment would be the sum of the heights of the

daily and semidaily tides at that moment. Therefore all we need do to determine the curve of the combined tide is to add the heights of the separate tides at various times throughout the day and draw a smooth curve through these plotted heights. The full-line curve of Figure 15 represents the combined tide.

We find, therefore, that the combination of a daily with a semidaily tide, under the conditions pictured in Figure 15. brings about a tide the features of which are decidedly different as regards morning and afternoon. In other words, this tide exhibits considerable diurnal inequality, both as regards the height of the tide and the time of the

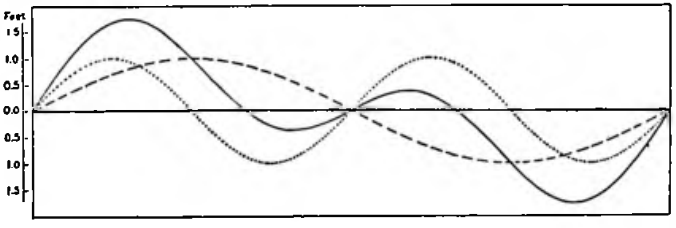


FIG. 15.—THE THREE TYPES OF TIDE, DAILY, SEMIDAILY AND MIXED

tide. This tide shows very close resemblance to the tide along the Pacific coast of the United States discussed in the preceding chapter.

The curves of Figure 15 represent, graphically, the three principal types of tide, which are known respectively as the semidaily, the daily, and the mixed type of tide. Wherever the tide shows two high and two low waters each tidal day, with but little difference between morning and afternoon tides, or more precisely with but little diurnal inequality, it is classed as a semidaily tide. At places where two high and two low waters occur each tidal day, but generally with considerable diurnal inequality, the tide is classed with the mixed type of tide. And where

the tide for a considerable part of the time shows but one high and one low water daily, it is classed with the daily type.

Now it is to be noted that the curves of Figure 15 represent the three types of tide in an idealized form. As we found in our consideration of the response to the tide-producing forces, the diurnal inequality depends on the moon's declination. When the moon is at its extreme declination, all tides exhibit *some* inequality; and similarly when the moon is at zero declination, the diurnal inequality becomes a minimum. Hence, when the tide at any place is assigned to a particular type, the reference is to the characteristics of the predominating tide at that place, or in other words to the characteristics which the tide at that place *generally* exhibits.

With this qualification in mind it follows that, of the tides discussed in the preceding chapter, those on the Atlantic coast of the United States belong to the semidaily type, those on the Pacific coast to the mixed type and the tides in the Gulf of Mexico to the daily type. On the coasts of Europe the semidaily type prevails; while on the shores washed by the Pacific and Indian Oceans the mixed type of tide predominates. The daily type is more restricted; beside the Gulf of Mexico, certain places in Alaska, in the Philippines, on the coast of China, and in other scattered localities may be mentioned.

In discussing types of tide it is of advantage to consider the tide as it actually occurs at any given place as being composed of two parts, a daily part and a semidaily part. Where the diurnal inequality is small, as in the semidaily type, it is evident that this is due to the existence of but a small daily part as compared with the semidaily part. Where the inequality is considerable, as in the mixed type, the daily and semidaily parts are of comparable magnitudes, and where the inequality becomes so large as to

eliminate one high and one low water in a day, as in the daily type, we shall find that it is due to the preponderance of the daily part.

It is obvious that there can be no very sharp line of demarcation between the semidaily type of tide on the one hand and the mixed type on the other. Between the two, there is a sort of twilight zone. The tidal mathematician, to be sure, lays down certain ratios between the daily and semidaily constituents of the tide to determine whether the tide at any given place is to be classed with the semidaily or mixed types. For our purpose, however, it will not be necessary to go into further detail regarding this matter, and we will content ourselves by defining the semidaily tide as one exhibiting little diurnal inequality, and the mixed tide as one exhibiting considerable inequality.

By assigning all tides that exhibit considerable inequality to the mixed type, it is evident that this type is made to cover a wide range, varying from those tides with just sufficient inequality to escape from the semidaily type to those with so much inequality that they are very close to the daily type. But even apart from the amount of inequality, the mixed type of tide shows three distinct varieties with respect to the phase of tide featuring the inequality, that is, whether the inequality is featured principally in the high waters or in the low waters or equally in both high and low waters.

The mixed tide shown in Figure 15 exemplifies the variety in which the diurnal inequality is featured in equal degree by both high waters and low waters. The difference in height between morning and afternoon high waters is the same as the difference in height between morning and afternoon low waters. Furthermore, the shift in time of the high waters is of the same amount and in the same direction as the shift in the low waters. And, while this

is an idealized form, there are a number of places where the diurnal inequality is featured in approximately equal amounts by both high and low waters. Hongkong, China, may be cited as an example.

The diurnal inequality in a mixed tide may, however, be featured principally in the low waters. In discussing the tides of the Golden Gate, attention was called to this feature of the tide, the percentage of diurnal inequality

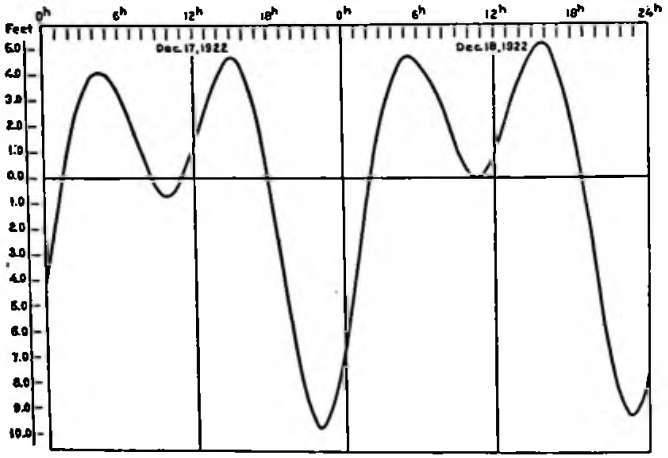


FIG. 16.—TIDE CURVE, SEATTLE, WASHINGTON

being, in the tropic tides, 87 per cent for the low waters and 33 per cent for the high waters. All along the Pacific coast the inequality is exhibited chiefly by the low waters, and in the waters of Puget Sound it becomes especially well marked. Figure 16 shows a good example of this, representing the rise and fall of the tide at Seattle, Washington, for two days, December 17 and 18, 1922. With an average range for the two days of about 9½ feet, the two high waters of each day differ by only one half of a

foot, while the two low waters of each day differ by 9 feet or more.

In some regions the diurnal inequality in the tides is featured principally in the high waters. It will be recalled that the little inequality found in the tides on the Atlantic coast of the United States is exhibited almost wholly by the high waters. But even in tides of the truly mixed type we find examples of this. Along the coasts of many of the islands of the Philippines, the diurnal inequality in the tides is featured principally in the high waters. This

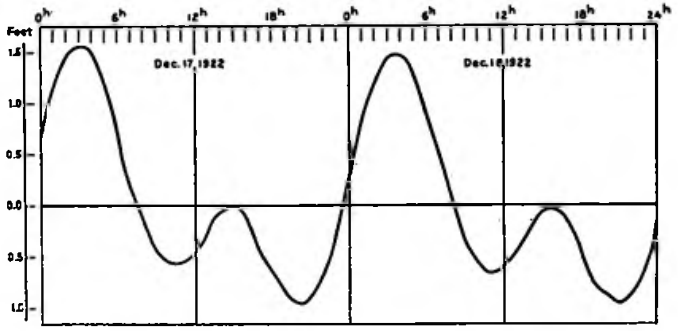


FIG. 17.—TIDE CURVE, HONOLULU, HAWAII

is true also with respect to a number of other islands in the northern Pacific, such as the Caroline Islands, to which the island of Yap belongs, and the Hawaiian Islands. South Australia may be cited as an example in another region of the world. The appearance of this variety of the mixed tide is well brought out in the tide curve for Honolulu for December 17 and 18, 1922, shown in Figure 17.

It will be noted that Figures 16 and 17 represent tide curves for the same days and a comparison of the two figures brings out the differences in the two varieties of the tide. The rise and fall of the tide at Honolulu is very much less than at Seattle. For the two days shown, the

average range at Honolulu was 1½ feet, but the two low waters of each day differed by less than half a foot, while the high waters differed by 1½ feet.

It can easily be shown that the different varieties of the mixed tide arise from different combinations of the daily and semidaily constituents. Turning back to Figure 15, we see that a daily and semidaily tide of the same range, the heights of which coincide at mean sea level, bring about a mixed tide in which the inequality appears in both the high and low waters. But the two constituent tides, even though of the same range, may combine in a number of different ways with respect to their phases. For example,

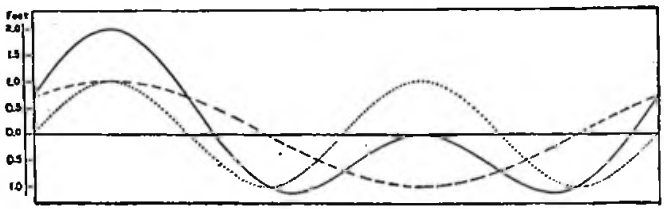


FIG. 18.—MIXED TYPE OF TIDE, INEQUALITY IN THE HIGH WATERS

instead of the two constituent tides coinciding in height at mean sea level, they may coincide at the time of high water, as shown in Figure 18. The combined tide now differs markedly from that of Figure 15, and exhibits in idealized form that variety of the mixed type of tide in which the inequality is featured by the high waters.

With the two constituent tides still of the same range, the coincidence in height may take place at the time of low water. To illustrate this case it will not be necessary to draw another figure. Simply turning the page upside down makes Figure 18 represent this case, if we bear in mind that the hours now run from right to left. And this combination, it is seen, gives us that variety of the mixed tide in which the inequality is featured by the low waters.

Thus far, in illustrating how different combinations of the constituent daily and semidaily tides give rise to the various forms of the mixed type of tide, we have assumed the ranges of these constituent tides to be the same. Several methods may be employed in resolving the tide actually occurring at any place into its constituent daily and semidaily parts. Of these methods the one known as harmonic analysis is the most powerful and the most satisfactory. But, as we shall take that method up later in connection with the prediction of tides, it will be sufficient here merely to mention that the analysis shows for different places varying relative ranges for the two constituent tides, as well as varying phases.

From the mixed type of tide in which the constituent daily and semidaily tides have equal ranges, there is therefore a gradual gradation on the one hand to the semidaily type in which the daily constituent becomes relatively small, and on the other hand to the daily type of tide in which the semidaily constituent becomes relatively small. From a mathematical study of the combination of cosine curves, it can be shown that when the range of the daily constituent is less than twice that of the semidaily, two high and two low waters will occur each day, the amount of inequality increasing with the relative increase of range in the daily constituent; when the range of the daily constituent is between two and four times as great as the semidaily, either two high and two low waters, or only one high and one low water will occur in a day, depending on the difference in phase between the two constituents; and when the range of the daily constituent is more than four times that of the semidaily, only one high and one low water will occur in a day.

The case in which the daily constituent has a range twice as great as the semidaily presents several interesting features. In Figure 19, the dotted curve represents a semi-

daily tide with a range of 2 feet and the dashed curve a daily tide with a range of 4 feet. The phases of these constituent tides are taken so that they both come to high water at the same instant. And when we form the combined tide we get a mixed tide—two high and two low waters in the tidal day—with the inequality in the high waters. By turning the page upside down, so that high waters and low waters are interchanged, we have the case in which the phases of the constituent tides are such that the low waters occur at the same instant. (It is important to note that the time now runs from right to left.) The

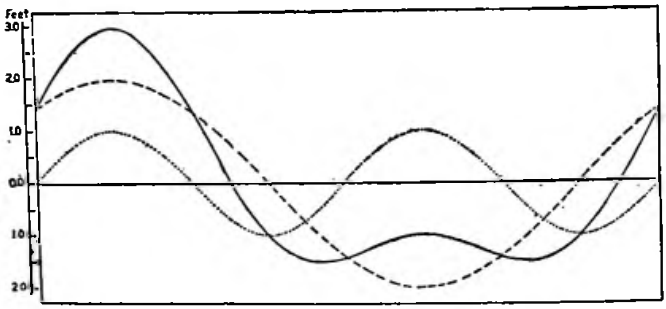


FIG. 19.—MIXED TYPE OF TIDE, RANGE OF DAILY CONSTITUENT TWICE THAT OF SEMIDAILY

combined tide is one of the mixed type, but with the inequality in the low waters.

If we make our constituent tides of Figure 19 combine under all the possible combinations of phases, we will find that in all cases but one, the combined tide will be of the mixed type, exhibiting varying degrees of inequality in the high and low waters. The limiting cases will be those shown in Figure 19, in which the inequality is either wholly in the high waters or wholly in the low waters. Between these there will be a great variety of tides with

varying degrees of inequality in both high and low waters, some with the inequality principally in the high waters, and others with the inequality principally in the low waters. One case was excepted in the statement above, and that one occurs when the phases of the two constituent tides are such that they are at mean sea level at the same instant.

In Figure 20, each of the constituent tides has the same range as in Figure 19, the dotted curve representing the

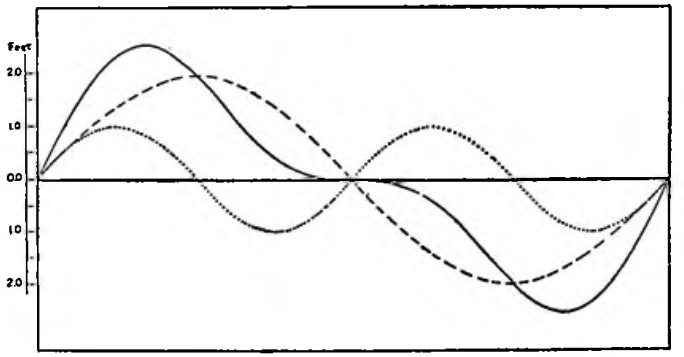


FIG. 20.—THE VANISHING TIDE

semidaily tide with a range of 2 feet and the dashed curve representing the daily tide with a range of 4 feet. The phases of the constituent tides, however, are now so taken that they are at mean sea level at the same instant. The tide that now results is shown by the full-line curve. In this tide there is but one high and one low water in a day, the other high and low water merging together to form a "stand" of several hours. This feature—the merging of a high and low water—is known as the vanishing tide.

The vanishing tide thus forms the connecting link between the tides of the mixed and daily types. In Figure

20 the vanishing tide occurred about mean sea level, giving a long stand at sea level, and this is the case with tides having the inequality in equal degree and in both high and low waters. With tides that exhibit the inequality principally in the high waters the stand comes below sea level and near the height of low water. With tides which show inequality principally in the low waters the stand occurs above sea level and near the height of high water.

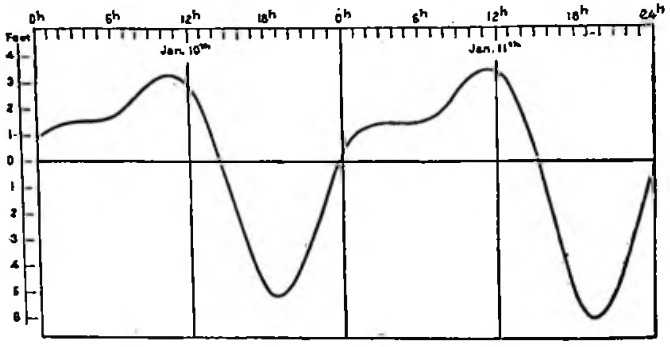


FIG. 21.—TIDE CURVE, PORT ANGELES, WASHINGTON, JANUARY 10-11, 1922

The relation between the position of the stand of the vanishing tide and the diurnal inequality becomes apparent. A glance at Figure 16 shows that where the inequality is principally in the low waters, the two high waters of a day are very nearly at the same height. Hence, if the higher low water is to merge with lower high water, it can do so only by rising to the height of lower high water, which height is also very nearly that of higher high water. Hence, the merging and therefore the stand must take place above mean sea level and near the height of high water. Figure 21, showing the rise and fall of the

tide at Port Angeles, Washington, in the Strait of Juan de Fuca, as recorded by an automatic tide gauge on January 11, 1922, illustrates this type of stand.

Similar reasoning in connection with Figure 17 shows why the stand of the vanishing tide in the case of tides featuring the inequality principally in the high waters must occur below sea level and near the height of low water. Figure 22, showing the rise and fall of the tide at Honolulu for April 11, 1913, exemplifies this variety of the vanishing tide. And it follows almost immediately that where the inequality exists in like degree in both high

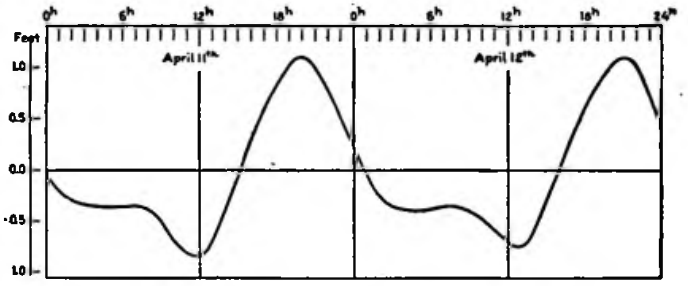


FIG. 22.—TIDE CURVE, HONOLULU, APRIL 11-12, 1913

and low waters the stand of the tide will occur midway between high and low water or near mean sea level.

The daily tide, as mentioned before, occurs where the daily and semidaily constituents have a ratio of 4 to 1 and may occur where the ratio is 2 to 1, depending on the phases of the daily and semidaily constituents. In the latter case the tide will, through the month, exemplify all three types of tide as shown in Figure 14 by the tides of Galveston. Here the daily constituent of the tide, as determined by harmonic analysis, is about 2½ times as great as the semidaily. When the moon is on the equator the daily tide-producing forces are at their minimum and at such

times the tide at Galveston resembles the semidaily type of tide. As the moon increases in declination, the tide takes on the appearance of the mixed type and finally passes over into the daily type.

At St. Michael, Alaska, the daily constituent of the tide is 3.8 times the semidaily and hence the tides are for the greater part of the time of the daily type. At Pensacola, on the Gulf coast of Florida, the ratio of the daily to the semidaily constituent is as 10 to 1, and the tide here, when

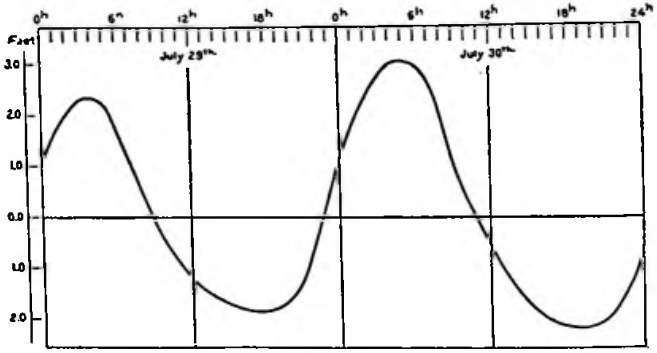


FIG. 23.—TIDE CURVE, ST. MICHAEL, ALASKA, JULY 29-30, 1898

not disturbed by wind or weather, is without exception of the daily type. At Do Son, in French Indo-China, the ratio is as 40 to 1 and the tide is always of the daily type. The appearance of a truly daily tide is well shown by the rise and fall of the tide at St. Michael, Alaska, for July 29 and 30, 1898. The curve looks much like that of the semidaily tide but with this distinction, that but one high and one low water occur in a day, high water and low water following each other by intervals of approximately 12 hours.

Our study of types of tide thus leads us to two conclusions: first, that these various types result from the inter-

action of the semidaily and daily constituent tides; secondly, that in the various seas these constituents have varying ratios to each other. In the Atlantic Ocean the semidaily constituent is the predominant one; in the Pacific and Indian Oceans the semidaily and daily constituents are present with comparable ranges, and in the Gulf of Mexico and in certain other smaller bodies of water the daily is the predominant constituent.

Furthermore, it appears that although the semidaily and daily constituents of the tide are brought about respectively by the semidaily and daily tide-producing forces of sun and moon, these constituent tides are not distributed in accordance with the distribution of their respective tidal forces. A glance at Figures 6 and 9 shows that the semidaily tide-producing forces are greatest in the low latitudes, while the daily forces are greatest in the high latitudes. Yet the Gulf of Mexico, in low latitudes, exhibits in its tides a preponderant daily constituent, while the tides of the North Atlantic Ocean, in the middle latitudes, show very little of the diurnal constituent. We are therefore led to conclude, too, that in the response to the tide-producing forces the various seas respond in different degree to the daily and semidaily tide-producing forces—in some cases responding better to the semidaily and in others to the daily. The reason for the differing behavior of the various seas to the same set of tide-producing forces must be left to the chapter devoted to the making of the ocean tide, where this aspect of the subject is of prime importance.

CHAPTER VI

RIVER TIDES

Rivers have always been of importance in the life of a people or a nation. In primitive societies the river constitutes a safe frontier for the small isolated communities, beside providing them with an abundant supply of water and the added food supply of fish. Advancing civilization in the invention of the boat gave the river an even greater rôle for it provided a highway for the exchange of materials and ideas and released man from the cramping environment of a restricted locality. With the enormous increase in world-wide trade characteristic of more recent times rivers have gained further in importance, especially the maritime portions, for these serve as highways to the seven seas.

The importance of the maritime portion of a river is attested to by the fact that of the great cities of the world located some distance from the sea, those situated on the maritime portions of rivers constitute a surprisingly large percentage. In the New World, Quebec on the St. Lawrence, New York on the Hudson, Philadelphia on the Delaware, Baltimore on the Patapsco, New Orleans on the Mississippi, and Buenos Aires on the Plata may be cited. In the Old World, London on the Thames, Liverpool on the Mersey, Hamburg on the Elbe, Rotterdam on the Maas, Antwerp on the Scheldt, Bordeaux on the Garonne, and Calcutta on the Hooghly are examples. And it is a significant fact that of these great cities, all but New Orleans are situated on tidal rivers. The river tide is thus a bond between these great cities and the open sea.

Now, it is obvious that because of the relatively restricted area occupied by the waters of a river, the direct effect of the tide-producing forces of sun and moon on the river waters is altogether negligible. Even in so large a body of water as Lake Superior the tide-producing forces of sun and moon bring about a tide with a range of but 2 inches. The tides in rivers are therefore due to the tides in the seas, with which they connect. And in its passage up a river the ocean tide is frequently modified to a very considerable extent, giving rise to phenomena that are characteristic of river tides.

The Hudson River, which empties into New York Harbor, furnishes us with an excellent example of a tidal river for our study of river tides. The ocean tide sweeps into New York Harbor past Sandy Hook; 14 miles from Sandy Hook the Hudson receives the impulse of the tide and carries it up along its course for a distance of 131 geographic miles, where it is stopped by the state dam at Troy. In tidal work it is more convenient to use the geographic or nautical mile than the statute mile, and for the rest of this chapter we shall therefore use the former in speaking of distances. The geographic or nautical mile has a length of 6,080 feet, while the statute mile has a length of 5,280 feet.

At its mouth, abreast of the Battery, New York City, the characteristics of the tide in the Hudson are very much like those of the tide of the open coast. Figure 24 shows the average rise and fall of the tide at the mouth of the Hudson, with reference to the moon's passage over the local meridian. High water and low water come respectively $8^h\ 15^m$ and $2^h\ 12^m$ after the moon's meridian passage, and it is to be recalled that with respect to the tide there are two meridian passages in a day, $12^h\ 25^m$ apart on the average.

A comparison of Figure 24 with Figure 11, representing

the rise and fall of the tide at Atlantic City, on the open coast, shows that the two tides are much alike. The range of the tide at the mouth of the Hudson is somewhat greater than at Atlantic City—4.4 feet against 4.0 feet—and there are slight differences in the amount of diurnal inequality; but like the tide at Atlantic City, the spring range of the tide at the entrance to the Hudson is 20 per cent greater than the average and the neap range 20 per cent less than the average. In but one respect is there a notable difference between the tides at the two places, and that is in the durations of rise and fall. At Atlantic City, the two

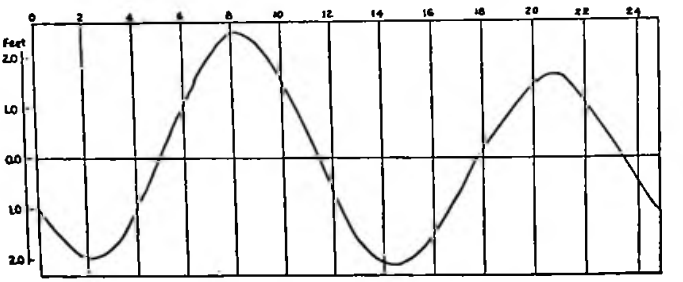


FIG. 24.—MEAN TIDE CURVE, MOUTH OF HUDSON RIVER

periods are practically the same, the average duration being $6^h\ 13^m$ for the rise and $6^h\ 12^m$ for the fall. At the mouth of the Hudson these periods are respectively $6^h\ 03^m$ and $6^h\ 22^m$. As distinguished from the open ocean tide, therefore, the period of rise has been shortened and the period of fall lengthened.

Since tides in rivers are due to the tides in the seas with which they connect, we may, in our study of river tides, disregard the tide-producing forces and consider the movement of the water from the viewpoint of wave motion. In its progress up the river we may therefore regard high water as the crest of the tide wave and low water as its

trough. From purely theoretical considerations of such wave motion, the mathematician has deduced that in a uniform channel the progress of the tide should vary with the depth, the formula being $r=\sqrt{gh}$ where r is the rate of progress of the tide, g the acceleration of gravity and h the depth of the waterway. The acceleration of gravity varies slightly from place to place, but its average value is 32.17 feet per second, and for practical purposes we may write the formula $r=5.67\sqrt{h}$. In this formula, r is expressed in feet per second and h in feet; however, since we are using the nautical mile for distance, it will be more convenient to express r in nautical miles per hour, which we can do by multiplying by 0.592 and hence our formula becomes $r=3.36\sqrt{h}$.

Returning now from our digression on wave motion, suppose we follow the tide up the Hudson and see what changes, if any, occur in its characteristics. Confining our attention to the time of tide, we will notice that as we go upstream the tide becomes gradually later, but with this distinctive feature: the times of high water and low water do not become retarded by the same amounts. Thus, at Tarrytown, 24 nautical miles up the Hudson, high water is $1^h\ 50^m$ later than at the mouth, but for low water the difference is $2^h\ 00^m$; at Storm King, 45 miles from the mouth, high water is $3^h\ 30^m$ later and low water $3^h\ 50^m$ later; and at Albany, 125 miles from the mouth, high water and low water are later by $9^h\ 30^m$ and $10^h\ 30^m$ respectively.

In its progress up the Hudson we may therefore say that the high water travels faster than the low water, and this is a characteristic feature of river tides. This may be explained on the basis of our formula given above, since at high water the tide wave has a deeper channel for its progress. As a consequence of the more rapid progress of high water, it follows that, as we go up the river, the

period from high water to low water becomes longer and between low water and high water shorter. In other words, the duration of fall is increased and the duration of rise decreased. At the mouth of the Hudson we found these periods to average $6^h\ 22^m$ and $6^h\ 03^m$ respectively; at Tarrytown they are $6^h\ 33^m$ and $5^h\ 52^m$; at Storm King, $6^h\ 41^m$ and $5^h\ 44^m$; at Albany, $7^h\ 21^m$ and $5^h\ 04^m$.

In consequence of the change in the periods of rise and fall, it is obvious that the appearance of the tide curve must change. Figure 25 brings out this change as exempli-

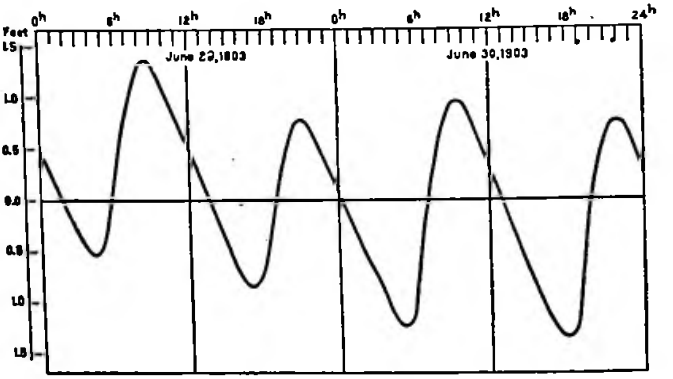


FIG. 25.—TIDE CURVE, ALBANY, NEW YORK

fied by the rise and fall of the tide at Albany on June 29 and 30, 1903. The tide curve here is no longer symmetrical with respect to high water, for the period of rise—from low water to high water—is short, while the period of fall is long.

The increase in the duration of fall and decrease in the duration of rise is characteristic of river tides. From general considerations this feature may be explained as due to the friction which the sloping bed of the river and the river current interpose against the advancing tide—di-

rected against the rising tide but coöperating with the falling tide. Of these two features of the river, the sloping bed is a fixed feature while the river current is variable, changing with the amount of water draining into the river. In periods of heavy rainfall or during freshets the current in the upper reaches of a river becomes greatly augmented and hence the progress of the tide at such times becomes impeded. In the Hudson River, during the spring freshets in March and April, the tide becomes completely obliterated at Albany and even for some distance below.

How far up a stream can the tide travel? Obviously this depends on the slope of the river bed and the velocity of the river current. In streams subject to considerable variation in the river current, the distance the tide will travel varies with the season. Under normal conditions the tide travels up a stream until stopped by falls or rapids. In the Hudson the tide travels 131 miles from the mouth, to be stopped by a dam. In the Mississippi River the tide becomes inappreciable not far from the mouth, through the fact that the tide in the Gulf of Mexico is small and the further fact of the enormous volume of water continually draining down the river. The greatest stretch of tided river is presented by the mighty Amazon. With a tide of a little more than 10 feet rising and falling at its mouth, the Amazon carries the tidal impulse upstream a distance of 450 nautical miles.

In traveling upstream the energy carried by the tidal impulse becomes lessened through the frictional resistance of the sides and bottom of the channel, and the work done against the river current. It follows, therefore, that in a channel of uniform width the rise and fall of the tide should decrease. And as a matter of fact observations prove this to be the case. However, the qualifying phrase "a channel of uniform width" must not be overlooked in this connection, for it is evident that in a channel, the

width of which contracts, the energy of the tidal impulse is concentrated into a smaller volume and may thus overcome the range-decreasing effects of friction.

Now most rivers are widest at their mouths and gradually decrease in width towards the head. But where this contraction is relatively small the range of the tide decreases from mouth to head. At the mouth of the Hudson the average range of the tide is 4.4 feet, while at Troy, the head of tide water, the range is 3.0 feet. The Delaware River likewise shows this gradual decrease in range. At Bombay Hook Point, where the river and bay may be considered as meeting, the range of tide is 6.0 feet; from this point the range of the tide decreases gradually, becoming 5.6 feet at Wilmington, 5.2 feet at Philadelphia and 4.2 feet at Trenton—the distance from Bombay Hook Point to Trenton being about 75 miles.

Three features thus characterize river tides: (1) The tide advances upstream at a rate dependent on the depth of the channel, the formula being approximately $r=\sqrt{gh}$; (2) in going upstream the duration of fall increases while the duration of rise decreases; (3) the range of the tide decreases from mouth to head. And while it is to be observed that these features are subject to modifying influences, they are the features that characterize the tides not only in rivers, but also in most bays and sounds. In Chesapeake Bay, for example, the tide exhibits features similar to those found in the Hudson River.

Attention was called to the fact that in a tidal waterway, the channel of which contracts rapidly, the concentration of the energy of the tidal impulse in a smaller space may overcome the frictional effects and result in a larger range upstream. This is exemplified in a number of funnel-shaped waterways. Delaware Bay at its mouth has a width of 10 miles, while at Bombay Hook Point the width is only 3½ miles, and observations show that at its

mouth Delaware Bay has a range of 4½ feet while at Bombay Hook Point—30 miles upstream—the range is 6 feet. A more striking case is presented by the tide in Bristol Channel. This waterway which makes a breach into the west coast of Great Britain is about 40 miles wide at its mouth but rapidly contracts, so that at the entrance to the Avon River, 80 miles upstream, its width is only 5 miles. At its mouth the average rise and fall of the tide is 20 feet while at Avonmouth it is 33 feet.

The Bay of Fundy may occur to the reader as an example of a funnel-shaped waterway. At its mouth this bay has a width of 75 miles, while at its head it forks into several small streams, and, as is well known, in one of these the greatest known rise and fall of the tide occurs. From less than 10 feet at its mouth the tide at the head of the Bay of Fundy acquires a range of almost 50 feet. And it was customary to ascribe this increase to the narrowing funnel form and shallowing bottom "cooping up the tide." But as we shall see later the increased range of tide at the head of the Bay of Fundy is due primarily to other causes; indeed, the tidal movement in this bay is of a totally different kind from the tide in the rivers and bays considered in this chapter. The same remark applies to Long Island Sound in which the range of the tide at the mouth is 2 feet while at its head it is 7 feet.

The effect of friction and of the river current on the advancing tide we have found to result in an increase in the period of fall and a corresponding decrease in the period of rise. In the tide curve, as exemplified in Figure 25, this exhibits itself as a steepening of the rise. Suppose now that the steepness of the tide curve during some portion of its rise be so increased that it becomes vertical. Obviously this means that for that portion of the rise the tide must come in as a wall of water. And here we have the explanation of the striking phenomenon known as the

bore. Where this occurs, the tide, for a portion of its rise, comes suddenly as a wall of water several feet in height. And from our consideration of the phenomena of river tides we should expect bores, wherever found, to occur in the upper reaches of a waterway where the resistance to the advancing tide is greatest. Furthermore, we should expect to find that the bore occurs about midway between low water and high water when the rise of the tide is taking place at its maximum rate.

Bores occur in a number of waterways, but in every case our deductions will be found to be correct. The bore in the Bay of Fundy, while not the most impressive one, has been carefully investigated and described by Dr. W. Bell Dawson, the Superintendent of the Canadian Tidal and Current Survey, and it is from his description that the following excerpts are taken.

This bore is best seen at Moncton, which is situated on the Petitcodiac River, one of the streams into which the Bay of Fundy branches at its head. At high tide the river at Moncton forms a sheet of water half a mile in width; while at low tide it is reduced to a stream about 500 feet wide running in a devious channel amongst bars and mud flats which now are exposed to view. The first sound of the approaching bore is very similar to the noise of a distant train heard across the water. It then increases to the usual hissing and rushing sound of broken water, but there is no mingling in this sound of any roar such as a waterfall makes when falling into deep water. About ten minutes after the first sound is heard, the bore arrives, advancing over the current of the river with a front of broken and foaming water which has a height of 2 or 3 feet.

The bore arrives, as we deduced, midway between low and high water or three hours before high water. The slope of the river, however, cuts off about half the rise of

the tide as shown in Figure 26, which represents the rise and fall of the tide as recorded by an automatic tide gauge at Moncton on August 14-16, 1920. The instantaneous vertical rise of from 2 to 3 feet is clearly brought out by the record, as well as the time relation of the bore to the time of high water. The effect of the river slope in cutting off about half the rise of the tide is shown by the very gradual fall from about 3 hours after high water to the time of arrival of the bore.

Bores have been noted in a number of streams. On the

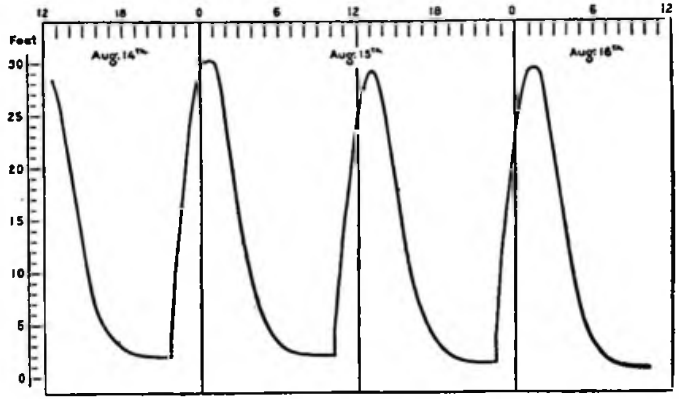


FIG. 26.—TIDE CURVE, PETITCODIAC RIVER, CANADA, AUGUST 14-16, 1920

American continents the bore has also been noticed in the Colorado River, at the head of the Gulf of California, and in several Brazilian rivers. On the other continents there are waterways that exhibit the bore, but probably the largest bore known is that found in the Tsientang Kiang, which is a Chinese river emptying into the China Sea. Here again we find the funnel-shaped waterway narrowing very considerably, and extensive shoals and mud banks. At Haining, where this bore appears well formed, it passes

as a wall of water of 10 feet or more, its front a sloping cascade of bubbling foam.

Another phenomenon sometimes exhibited by tides in rivers is the so-called reversible tidal waterfall. An excellent example of this is found on the St. John River in the Canadian Province of New Brunswick. The St. John debouches into the Bay of Fundy through a comparatively wide channel; but a short distance from its mouth the river channel becomes a narrow rocky gorge which further upstream expands into a large basin. At the mouth of the St. John the tides have a range of over 20 feet, and when the tide is rising at its greatest rate midway between low water and high water, the narrow gorge does not permit sufficient passageway for the large volume of water. Hence, at this time of the incoming tide the water rises higher on the ocean side of the gorge and a waterfall is formed in the upstream direction. This continues until the tide has come to high water and started falling, when the water on the two sides of the gorge is at the same level.

From this time, however, the water on the upstream side of the gorge cannot fall as rapidly as on the ocean side and hence a higher level now results on the upstream side of the gorge, with a waterfall in the downstream direction. And this continues until low water has been passed on the ocean side of the gorge and the water has started to rise, when the level at the two ends of the gorge is again the same. The tide thus here forms a waterfall flowing alternately six hours in the upstream and downstream directions. At its extreme the difference in the level of the water between the two ends of the gorge is from 6 to 12 feet.

In our consideration of the movement of the water brought about by the tide-producing forces of the moon and sun, we have thus far confined our attention exclusively to the rise and fall, that is, to the vertical motion of the

water. In the movement of tidal waters, however, it is obvious that great masses of water are transported for considerable distances—onshore and offshore in the seas, and upstream and downstream in bays and rivers. This transport of the water gives rise to a horizontal motion—the so-called tidal current—that accompanies the rise and fall of the tide, and it is to a consideration of the tidal current that we must now proceed.

CHAPTER VII

TIDAL CURRENTS IN RIVERS AND BAYS

THE title of Chapter IV, The Response to the Tide-Producing Forces, should perhaps have had a subtitle appended to indicate that only the vertical motion of the ocean waters was to be considered in that chapter. For the response of the ocean waters to the tide-producing forces of sun and moon actually manifests itself in two ways: in a vertical rise and fall and in a horizontal forward and backward movement. These two movements are intimately related, being merely different features of the same phenomenon. And it is only for convenience of study that they are discussed separately.

Of the two movements of the waters brought about by the tide-producing forces of sun and moon, the vertical movement is called the tide and the horizontal movement is called the tidal current. Obviously, it would be of advantage to have some distinctive term with which to designate the whole phenomenon that includes both tides and tidal currents. Unfortunately, however, we have no such term and we are compelled to use "the tide" or "the tides" or even "flood and ebb" to denote the phenomenon in its general sense.

In navigation, the vertical rise and fall of the tide is a matter of prime importance in the immediate vicinity of the coast or in areas of shoal water. Out in the open sea, however, where the navigator is sure of sufficient depth, he disregards entirely the vertical rise and fall, but the hori-

zontal movement of the water now becomes a matter of practical importance. Moreover, out in the open sea the rise and fall of the tide is no longer obvious. Hence, the mariner frequently speaks of the "tide" when it is the "tidal current" that he has in mind; as when he says "the tide is ebbing."

It is essential, however, to distinguish clearly between tide and tidal current, for the relation between the two is not a simple one nor is it everywhere the same. At one place we may find a strong current accompanying a tide having a moderate rise and fall, while at another place a like rise and fall of the tide may be accompanied by a very weak current. Furthermore, the time relations between current and tide at different places are subject to wide variation. To avoid confusion, therefore, we shall use "tide" in reference to the vertical movement of the water and "current" in reference to the horizontal movement.

Not only must the tidal current be distinguished from the tide, but it must also be carefully distinguished from nontidal currents. In brooks and rivers we are familiar with the current that carries the water from the land to the sea. This current is always seaward and is due to gravity, not to the tide-producing forces; it therefore does not partake of the characteristics of tidal currents. Winds blowing steadily for a number of hours over extended bodies of water give rise to currents, but these currents likewise, being due to nontidal causes, differ from tidal currents. In the open sea, there are great streams—rivers in the sea—like the Gulf Stream in the Atlantic and the Kuro Siwo in the Pacific which form part of the system of oceanic circulation brought about by winds and differences in temperature and density of sea water. These great currents, too, being brought about by nontidal agencies, belong to the class of nontidal currents, the char-

acteristics of which are different from those of tidal currents.

It is to be noted, however, that in the open ocean and in inshore tidal waters, tidal currents and nontidal currents occur together, the actual current experienced at any time being the resultant of the interaction of the two classes of currents. In some places tidal currents predominate and in others nontidal currents predominate. In the open sea tidal currents are generally very weak; along the coast they become more noticeable, and in narrow entrances to bays and in constricted parts of rivers they frequently attain very considerable velocity. We shall therefore begin our study of tidal currents with their characteristics as manifested in bays and rivers.

Having used the Hudson River for our study of river tides, it will be of advantage to use this river likewise for our study of tidal currents, since this will permit us to determine the relation of the current to the tide. Furthermore, we have in the Hudson River for our purposes an excellent series of current observations made in 1922. In the summer of that year the Coast and Geodetic Survey, in coöperation with the Engineer Office of the War Department, carried out a comprehensive current survey of New York Harbor. During this survey current observations were made in the Hudson at a number of points, the observations being made from small barges anchored in the stream. On the 15th of August, 1922, let us imagine ourselves on board the barge anchored in the center of the channel abreast the entrance of the Hudson River, and let us assume that we had made arrangements to observe the velocity and direction of the current every half hour, beginning at midnight or zero hour of the 15th.

But how are we to observe the velocity of the current? We want our results to be accurate measurements and not merely estimates. There are a number of methods for

observing currents, but one of the simplest is by means of a current pole and log line. The current pole is an ordinary pole about 15 feet long and about 2 inches in diameter. It is weighted at the bottom with sufficient sheet lead to submerge in the water all but 1 foot of its length, and to this upper foot is attached a graduated line—the so-called log line. The line is graduated in such wise that for an observation interval of 60 seconds it will give the velocity of the current in nautical miles per hour or, as it is technically known, in knots.

Our observations are to be made exactly on the hour and half hour. Hence, a few minutes before zero hour of the 15th, we see to it that our current pole is in the water, with the log line held fast between our fingers. Promptly on the stroke of the hour we allow the pole to go with the current, paying out the log line as fast as the pole carries it away. Exactly 60 seconds after the release of the current pole, the log line is held fast and the distance run off read. We find this to be 0.35 knot, and we therefore note this in our record book and also the direction of the current which was upstream. Every half hour thereafter we repeat the operation until the clock strikes midnight. During this period, the current has flowed alternately upstream and downstream, both morning and afternoon.

We are now in possession of a series of figures giving the velocity and direction of the current for every hour and half hour for the entire day of August 15, 1922. To help us in visualizing the matter, we will do as in the case of the tides—plot the observed values on cross-section paper. However, before we do this, let us note that in plotting tidal observations we are concerned with two factors only, time and height; whereas in the matter of currents we are dealing with three factors, time, velocity, and direction. We must therefore adopt a somewhat different procedure in plotting our current observations. A very convenient

method consists in plotting the velocities of the current in the upstream direction above the zero velocity line, and the downstream velocities below this line. If we do this and then draw a smooth curve through our plotted points the result is the upper curve of Figure 27.

It is customary to speak of the upstream direction of the current as flood, and of the downstream direction as ebb. In our current curve, therefore, the curve above the zero velocity line represents the flood current, while the curve below this line represents the ebb current. For

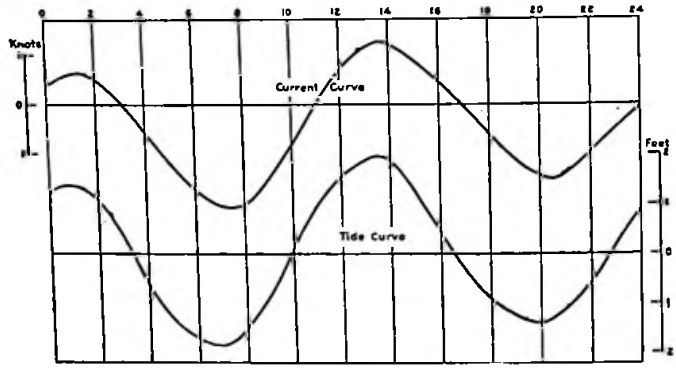


FIG. 27.—CURRENT CURVE AND TIDE CURVE, HUDSON RIVER ENTRANCE, AUGUST 15, 1922

purposes of comparison the rise and fall of the tide at the entrance to the Hudson is shown in the lower curve of Figure 27. And we are immediately struck with the close resemblance between the current and tide curves. But before taking up the relation between current and tide, let us examine in greater detail the characteristics of the current as shown in our figure.

To begin with, we notice that during any flood or ebb period the current does not flow with a constant velocity. At the instant when the current changes in direction from

flood to ebb or vice versa it is obvious that there is no horizontal movement of the water; in other words, the velocity of the current at that instant is zero and this phase of the current is known as the "slack of the current" or "slack water." After slack water the velocity of the current increases gradually for a period of about 3 hours, when the maximum velocity is attained, this being known as the "strength of the current"; for the flood period this is called strength of flood and for the ebb period the strength of ebb. From the strength of the current, the velocity of the current decreases gradually for another period of about 3 hours, when slack water is again reached, and the current then reverses its direction and goes through a similar cycle of about 6 hours in that direction. The curve of the current like that of the tide approximates the form of the cosine curve.

If we continue our current observations for a number of days, we will find that the current curve from day to day reproduces, in large measure, the features of the tide curve. At times of new and full moon, when the so-called spring tides occur, the current shows an increased velocity, and when the moon is in her first or third quarter, the decreased ranges of the neap tides are accompanied by relatively weak currents. Similarly, perigean tides bring current velocities greater than the average, while apogean tides bring current velocities less than the average. In fact, from theoretical considerations it can be shown that changes in the moon's phase and distance bring about changes in the velocity of the tidal current which are proportional to the corresponding changes in the range of the tide.

It will be noted in Figure 27 that the two floods of the day differ somewhat, as also do the two ebbs, and that these differences resemble respectively the differences in the high waters and the low waters of the tide. That is, the current,

like the tide, exhibits diurnal inequality. But in general the diurnal inequality in the current at any place is about half that of the tide. That this should be so can be shown without much difficulty by the use of mathematics, but here we need not go into such details except to state the general principle involved, which is, that variations in the range of the tide brought about by the semidaily tide-producing forces are reflected in the velocity of the current in the same proportion, while variations in range brought about by the daily forces are reflected only half as much in the velocity of the current.

In everyday use, the flood current is frequently taken as synonymous with the rising tide and the ebb current as synonymous with the falling tide. This is exemplified in the lines of the poet:

There is a tide in the affairs of men
Which taken at the flood, leads on to fortune.

But, if we examine the time relations of current and tide as shown in the curves of Figure 27, we see that in the entrance to the Hudson River on the first half of the flood the tide is rising, while on the second half the tide is falling. Similarly, on the first half of the ebb the tide is falling while on the second half it is rising. And this is the relation of current to tide in the entrance to most bays and rivers.

In the year 1922 the current observations at the entrance to the Hudson were continued for more than a month. The average strength of flood was found to be 1.2 knots and the average strength of ebb 2.2 knots. The average duration of flood was found to be $5^h\ 02^m$, while the duration of ebb was $7^h\ 23^m$. The time relations of the current to the tide were found to be as follows: Slack before flood $2^h\ 20^m$ before high water; strength of flood at high water; slack before ebb $3^h\ 40^m$ before low water,

and strength of ebb 0^h 10^m after low water. We shall consider later the reasons for the greater velocity and greater duration of the ebb current, but here we want to note the fact that the slack of the current in the entrance to the Hudson River comes midway between high and low water, while the strength of the current comes at the times of high and low water.

If we went up the Hudson River and made observations of the current and tide, say every ten miles, we would find that while the current in its essential features was much like the current at the mouth, except that it came later upstream, the time relations of current and tide would be found to be gradually changing. If we consider what happens near the head of tide water we shall see the reason for this change.

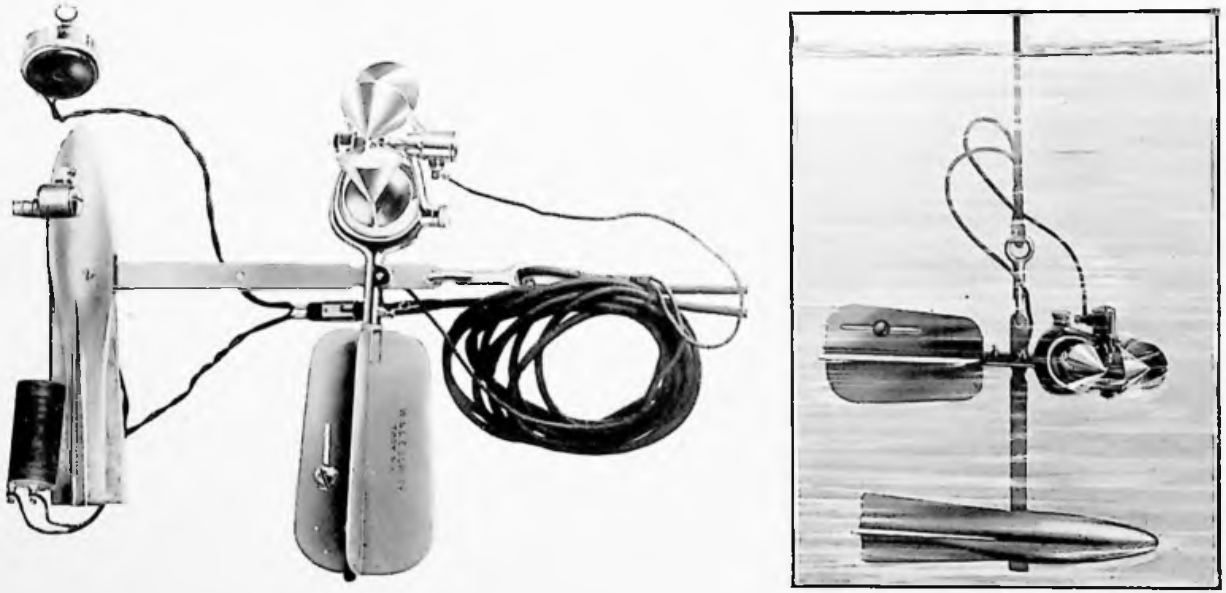
Fixing our attention on a small stretch of the river near the head of tide water, it is obvious that as long as the current is flooding, water is being brought into this area. The tide in this stretch must therefore be rising, and it can stop rising only when the current stops flooding. In other words, at the head of tide water the slack of the current comes at the time of high water, while at the entrance the slack comes midway between high and low water. And similar reasoning will show that at the head of tide water the slack before flood comes at the time of low water. From the entrance to the head of tide water, therefore, the relation of current to tide changes by three hours and this change takes place gradually. Likewise, at the head of tide water the current is running at its strength when the height of the tide is changing most rapidly; that is, midway between high and low water, while in the entrance the strength of current occurs at the times of high and low water.

The results which we arrived at in the preceding paragraph from general considerations are found to be the facts

from actual observations; and as we go up the Hudson we find that at West Point, 50 miles above the mouth, the strength of the current occurs three-quarters of an hour before time of tide; at Poughkeepsie, 75 miles above the mouth, 1½ hours before the time of tide; at Hudson, 115 miles above the mouth, 2¼ hours before the time of tide, and at Albany, which is near the head of tide water and 145 miles above the mouth, the strength of current comes about 3 hours before local high or low water.

Now it is to be noted that the current pole which we used for measuring the velocity of the current extended to a depth of 14 feet below the surface. The question naturally arises as to what takes place below that depth. Or, looking at it from another point of view, we may ask, to what depth does tidal action extend? But before these questions can be answered, there is the question: How are the velocities of the current at considerable distances below the surface measured? For, obviously, the current pole method is not practical for considerable depths.

The current meter furnishes a very convenient instrument for measuring the velocity of subsurface currents. Various forms of this instrument are in use, Figure 28 representing the Price Current Meter. In principle, it is very simple. Essentially, it consists of 6 cups equally spaced in a horizontal plane that rotates on a vertical axis so that they can respond only to motion in a horizontal plane. In action, the meter is connected with a battery and with an ordinary telephone receiver. When the meter is lowered to any desired depth, the current impinging on the cups causes them to revolve and each complete revolution is indicated to the observer by a tick in the telephone receiver, so that the number of revolutions in a given interval of time, usually 60 seconds, can be counted. The current meter is calibrated so that one revolution corresponds to a definite velocity of the current and it is there-



Courtesy of W. L. E. Gurley

FIG. 28. THE PRICE CURRENT METER

fore a simple matter to determine the velocity of the current at any desired depth.

By means of the current meter current observations have been made at various depths in tidal waterways, from the surface to the bottom. And without going into detail it may be said that the tidal movement extends from the surface to the very bottom, the velocity being practically the same from the surface to the bottom. Furthermore, at all depths the characteristics of the current are similar, though it must be stated that in bays and rivers which drain large areas the effect of the drainage water is to bring about certain differences in the tidal current near the surface which we must now consider.

It will be recalled that the observations with the 14-foot current pole at the entrance to the Hudson River gave for the flood current a velocity at strength of 1.2 knots and a duration of $5^h\ 02^m$, while the ebb current had a strength of 2.2 knots and a duration of $7^h\ 23^m$. This means that for a layer 14 feet deep the water was running out for a longer period and with a greater velocity than it was running in, or, in other words, a greater volume of water went out on the ebb than came in on the following flood. This fact gives us a clue to the factors involved, for in so far as the water brought in by the tide is concerned, it is obvious that the outflow cannot be greater than the inflow; hence, the excess of outflow must be nontidal water, and this excess is the water that drains off the land and finds its way through rivulet and stream into the Hudson to be carried to the sea.

Now the ocean water that the tide carries in is salty and therefore heavier than the nontidal water that drains from the land. Hence, in a river or bay the nontidal water tends to remain near the surface and as this nontidal water is setting toward the sea, it must increase the velocity of the ebb current by its own velocity and decrease

the velocity of the flood current by the same amount; and, as a further consequence, it increases the duration of ebb and decreases the duration of flood. It is evident, too, that in periods of heavy rainfall or during freshets the volume of nontidal water setting downstream may be so great as to give rise to a velocity greater than the velocity of the tidal current at strength of flood. At such times, therefore, the flood current is completely overcome, and for a period of several days we may have the current setting seaward at all times. This is frequently exhibited in the upper reaches of tidal streams draining considerable areas like the Hudson or the Potomac Rivers.

During the period of flood, an object floating in the water is carried upstream while on the ebb it is carried downstream. How great is the distance in each case? If the velocity of the current were constant throughout a flood or ebb period, the distance an object floating in the water would be carried is obviously the velocity multiplied by the time. However, the velocity of the tidal current is not constant but varies as the cosine curve. During a flood or ebb period, therefore, the distance a floating object is carried upstream or downstream equals the average velocity of the current during this period multiplied by the period.

There are several methods for determining the average velocity of the current during a flood or ebb period. One method consists in measuring the current curve at frequent intervals, say every five or ten minutes, and taking the average of all these measurements. But a much simpler method consists in making use of a mathematical property of the cosine curve, which is that its average height is 64 per cent of its greatest height.

The average duration of a flood or ebb period is $6^h\ 12\frac{1}{2}^m$ or 6.21 hours. Hence, for a current which has a velocity at strength of one knot, a floating object would be carried

a distance of $6.21 \times 0.64 = 3.97$ nautical miles or 24,000 feet. Where the flood and ebb velocities and also the periods of flood and ebb are equal, a floating object would be carried downstream a certain distance on the ebb and upstream the same distance on the flood. But all bays and rivers carry the drainage waters from more or less extended regions and hence the velocity and also the duration of ebb is greater than the velocity and duration of flood; objects floating in the water are therefore carried downstream further on the ebb than upstream on the flood, and thus finally find their way out to sea. This is of very considerable importance in the disposal of the sewage from communities situated on tidal streams.

How great a velocity may a tidal current attain? In the Hudson, we found a velocity up to 2 knots or more; in the Golden Gate, the entrance to San Francisco Bay, the velocity of the tidal current is from 3 to 4 knots; in Hell Gate, New York Harbor, 5 to 6 knots; in Seymour Narrows, British Columbia, 8 to 10 knots; and, at certain other places it may go up as high as 12 knots.

The features of the current found in the Hudson River are the characteristic features of the tidal current in most bays and rivers. Indeed, with the exception of the relation of time of current to time of tide, we may say that the features of the current in the Hudson are characteristic of the tidal current in all bays and rivers. There are, however, some bodies of water in which the time relations of current and tide are different than in the Hudson. Long Island Sound and the Bay of Fundy are examples.

In Long Island Sound, observations show that from the mouth to the head the current changes but little in time, notwithstanding the fact that the Sound has a length of over 90 miles. Furthermore, over the greater part of this body of water, the relation of time of current to time of tide is, within narrow limits, constant; the strength of the

current comes midway between high and low water, while the slack of the current comes near the times of high and low water.

If now we examine the tides in Long Island Sound, we find that over the greater part of the 90-mile stretch the tide changes but little in time, high water and likewise low water occurring very nearly simultaneously throughout the Sound. And what is even more striking about the tides here is the fact that instead of decreasing in range from mouth to head, there is actually an increase. At the entrance to Long Island Sound, the tide has a rise and fall of 2.5 feet, while at its head the rise and fall is 7.2 feet.

In Long Island Sound, therefore, the tidal movement—comprehending under this term both the tides and the currents—differs from the movement found in most bays and rivers. It is therefore necessary for us to consider now the nature of tidal movements and the precise relation of current to tide. But before doing so there is a matter connected with tidal currents that had perhaps better be discussed at this time, and that is the question of the duration of slack water.

Where the current at time of strength attains a velocity of 2 knots or more, it becomes a matter with which the navigator must reckon; and where a velocity of 5 knots or more exists, it frequently becomes necessary for the navigator to await the time of slack water before attempting to make the passage. The question is therefore frequently raised: How long does slack water last?

To this seemingly simple question no direct answer can be given, for it all depends on just what is meant by the term "slack water." A glance at the current curve of Figure 27 shows that the time during which the current has no movement is but a moment—the instant when the current curve cuts the zero velocity line—and hence when the tidal mathematician speaks of slack water he has in

mind that mathematical instant. Apart from the mathematician, however, slack water is thought of as a period of time extending over a number of minutes during which the current is at a standstill, neither flooding nor ebbing. And the layman is at a loss to understand why no direct answer can be given to his simple question: How long does slack water last?

But if we consider the period of slack water to be the interval of time during which the velocity of the current is so small that for *practical purposes* it may be disregarded, we make the term connote the meaning attached to it in ordinary usage. A current with a velocity less than one-tenth of a knot may for all practical purposes be disregarded; indeed, it is difficult to measure so small a velocity either with current pole or with current meter. We may therefore give the term slack water a definite meaning in ordinary usage by defining it to be the period during which the velocity of the current is less than one-tenth of a knot.

With this definition we are in a position to answer the question: How long does slack water last? For all that need be done is to measure, on the current curve for any given place, the interval during which the velocity of the current is less than one-tenth of a knot. For example, from the current curve in Figure 27 we find that slack water at the mouth of the Hudson lasts about 15 minutes.

It is obvious that with strong currents the duration of slack water is relatively short while with weak currents it is greater. In fact, since the current curve approximates closely the cosine curve, it can be shown that the duration of slack water varies as the strength of the current. And regarding the current curve as a cosine curve we may compute the duration of slack water directly from the equation of the cosine curve. With currents which at strength have a velocity of 1 knot, the duration of slack

water, that is, the period during which the velocity of the current is less than one-tenth of a knot, is 24 minutes; with currents of 2 knots' strength, 12 minutes; 3 knots, 8 minutes; 4 knots, 6 minutes; 5 knots, 5 minutes; 6 knots, 4 minutes; 8 knots, 3 minutes; 10 knots, 2 minutes.

CHAPTER VIII

THE RELATION OF CURRENT TO TIDE

In the preceding chapters, the rise and fall of the tide and the flood and ebb of the current were discussed as separate phenomena. That, however, was done merely as a matter of convenience; for our consideration of the matter has led us to conclude that the two movements—the vertical rise and fall of the tide, and the horizontal forward and backward movement of the current—are intimately related, constituting, in fact, the single phenomenon of tidal movement. We therefore find ourselves now confronted by the questions of what precisely is the nature of tidal movement, and just how are tide and current related to each other.

These questions lead us directly into the subject of wave motion, a subject so very extensive that an adequate discussion would require a volume of its own. And while wave motion is a very interesting subject, we want to consider here only so much of it as is necessary for an understanding of tidal movements. Briefly, therefore, we may say that any body of water which is disturbed from its condition of rest tends to set up waves. Examples are numerous in nature: a pebble dropped into the pond, the wind disturbing the surface of the lake, the boat propelled through the water—all these give rise to waves. And in the same way the tidal forces of moon and sun, acting upon the waters of the sea bring about the slow but mighty wave which stirs the sea to its very depths and which we call, for short, the tide.

There are various kinds of waves, each having its own laws of behavior. In tides, however, we need concern ourselves only with two kinds, known respectively as progressive waves and stationary waves. On a very much reduced scale we may study the characteristics of each of these varieties of waves as manifested in a tank of water.

Suppose we have a rectangular tank, say 20 feet long, filled with water to a depth of about half a foot. From a condition of rest this water may be made to sustain two entirely different types of wave motion. In the first place, by agitating the water at one end with a paddle we may start a wave which will advance or progress from one end of the tank to the other, the outline of this wave resembling the cosine curve. A wave of this kind is called a progressive wave. We may also set up in our tank of water a wave motion of an entirely different kind by raising and then immediately lowering one end of the tank. The water will then swash alternately from one end of the tank to the other about an axis situated in the middle of the tank, and this kind of wave motion is known as stationary-wave motion. These two types of wave movement are exemplified in the movements of tidal waters and we shall now proceed to consider each of these types in greater detail.

The outline of a progressive wave resembles the cosine curve shown in Figure 29. When a wave of this character traverses a body of water, it has been found both from theory and from observation that its rate of advance depends on the depth of water, the formula being $r=\sqrt{gh}$ where r is the rate of advance of the wave, g the acceleration of gravity, and h the depth of water. In this connection it is to be recalled that in our discussion of river tides in Chapter VI, the progress of the tide up a river was expressed by the same formula. Indeed, the tide in most bays and rivers may be regarded as progressive waves

entering the mouths of these waterways and traveling upstream.

It is important to note, however, that in progressive-wave motion we must distinguish carefully between the movement of the wave form and the movement of the water particles themselves. In the progressive wave in our tank of water, for example, it might appear at first glance that the movement of the wave consists of the bodily movement, from one end of the tank to the other, of a quantity of water deformed into the outline of a wave. But if we place some floating object in the water, a cork for instance, it becomes obvious at once that the movement of the wave is the movement through the water of a certain *form* or *arrangement* of the water particles, and this movement is

FIG. 29.—OUTLINE OF PROGRESSIVE WAVE

from one end of the tank to the other. The movement of the cork, which is the movement of the water particles themselves, is seen on examination to consist of a movement for a short distance in the direction of the wave while it is on the crest of the wave, and of a movement for a like distance backward when it is in the hollow of the wave.

If we regard Figure 29 as representing a vertical section through our tank of water, the line AB will represent the undisturbed level of the water and the curve will represent the outline of the progressive wave. On careful examination it will be found that as the wave passes from one end of the tank to the other, the cork floating in the water goes through a cyclic movement that has both a horizontal and a vertical component. Vertically it rises and falls with

the wave, being highest when it is on the crest of the wave and lowest when it is in the trough of the wave. If we plot this rise and fall with regard to time the result will be a cosine curve. Horizontally, the cork moves forward, or in the direction of the wave, when it is above the line AB, and backward when it is below. The change in direction, therefore, must take place when the cork is at the level of the undisturbed water. And if the horizontal velocity of the cork is plotted as a function of time, the result will be a cosine curve, too.

It will be noted that the characteristics of progressive-wave motion as exhibited in a tank of water resemble closely the characteristics of the tidal movement in the entrance to the Hudson discussed in the two preceding chapters. And as in the progressive wave in our tank of water, so in tidal movement the vertical and horizontal movements are intimately related parts of the same phenomenon. In our daily lives, however, we are so familiar with the movement of water from a higher to a lower level that frequently the statement is made that the tidal current is due to a difference in level brought about by the rise and fall of the tide.

That this statement is incorrect is evident from the following considerations. If the horizontal movement in a progressive wave were due to differences in level, we should expect this horizontal movement to have its greatest velocity at the steepest portion of the wave, or at points C and E in Figure 29. Likewise the velocity should be least at points D and F where the slope of the wave is zero. But as a matter of fact we have found that conditions are exactly the reverse, and therefore the horizontal motion in a progressive wave is not due to differences in level. Indeed, it is much easier to explain the vertical rise and fall in wave motion as a consequence of the horizontal movement, but that would lead us into too detailed a consideration of wave motion.

We may therefore sum up our discussion of progressive-wave motion, in so far as it relates to tides, by saying that in most bays and rivers the tide enters as a progressive wave, and from the nature of this type of wave movement, strength of the current comes at the times of high and low water, while the slack of the current comes midway between the times of high and low water. The changes that occur in the time relations of tide and current as the tide progresses upstream are brought about by friction and by the gradually shoaling bottom.

Taking up now the consideration of stationary-wave motion, we find an entirely different kind of movement.

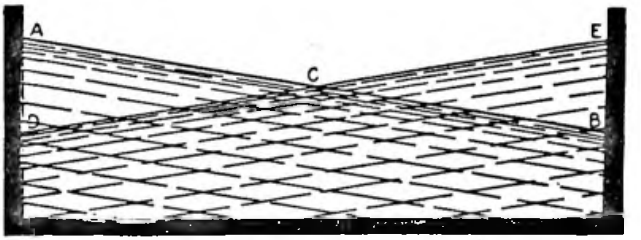


FIG. 30.—STATIONARY WAVE

Schematically we may represent this type of wave movement by Figure 30. From a condition of rest we may start a stationary wave in a tank of water by raising and immediately lowering one end of the tank. The water will then swash from one end of the tank to the other in such wise that at the instant the motion towards the left ceases, the surface of the water will be in the position represented by the line AB. In other words, it will be high water for half the tank from A to C at the same instant that it is low water for the other half of the tank from C to B. The water then begins its oscillation in the opposite direction, at the end of which the surface assumes the position represented by the line DE, when it is low water for half the

tank from D to C at the same instant that it is high water for the other half of the tank from C to E.

Here, too, it is to be noted, although the wave movement is entirely different from progressive-wave movement, there is a vertical and a horizontal component. And if we should measure these vertical and horizontal movements separately and plot them as functions of time we would find the curves to be cosine curves.

Examining the characteristics of stationary-wave movement, it is seen that the horizontal motion must cease at the same instant as the vertical motion. Applying tidal terms to the horizontal and vertical movements of stationary-wave motion, we may say that the slack of the current comes at the time of high and low water. It follows, therefore, that the greatest horizontal velocity or strength of the current comes midway between the times of high and low water, which, it is to be recalled, is exactly the reverse of conditions in progressive-wave movement.

From Figure 30 it is evident that in stationary-wave movement the amount of rise and fall varies in proportion to the distance from the axis about which the water oscillates. At the axis itself the vertical rise and fall is zero, while at the ends of the tank it is at its maximum. Furthermore, it is obvious that it is high water at the same instant over half the tank from A to C, and at the same time it is low water over the other half from C to B. Likewise the horizontal movement ceases at the same instant all over the tank. In all these features, therefore, the stationary-wave movement differs from progressive-wave movements.

The characteristics of tides and currents in bays and rivers, discussed in the previous chapters, prove the tidal movement in these bays and rivers to be primarily of the progressive-wave type. Is there any body of water which exhibits the characteristics of stationary-wave movement?

Let us examine the tidal and current phenomena in Long Island Sound, which is the eastern entrance to New York Harbor.

Long Island Sound is a body of water having a length of about 80 nautical miles and a depth averaging about 65 feet. At its eastern end where it receives the tide from the ocean, the rise and fall or range of the tide is about 2½ feet, while at its western end, where it leads into East River, the rise and fall is very nearly 7½ feet. The Sound is shaped somewhat like a double convex lens, and hence the relatively great increase in range at the western end might be ascribed to the converging shore line. But it is to be noted that the range of the tide increases in a fairly uniform manner from the eastern end despite the fact that at its middle the Sound has a greater width than at the ocean entrance. From 2½ feet at the ocean entrance the range of the tide in Long Island Sound increases to very nearly 6 feet at its middle, where it has its greatest width.

Another peculiar feature of the tide in Long Island Sound is that there is but little difference in time of high water or of low water from one end to the other; that is, over the greater part of its area the tide is very nearly simultaneous. Thus from Cornfield Point Light Vessel near the ocean entrance, to Execution Rocks near the East River entrance, a distance of about 65 nautical miles, the tide becomes later by barely half an hour. Here the tide evidently does not move as a progressive wave, for such a wave would require a much greater period of time to traverse the 65 miles, as we can easily calculate. In Chapter VI, the formula for the rate of advance of a progressive wave, in nautical miles per hour, was found to be equal to $3.36\sqrt{h}$ where h is the depth of the waterway. In a depth of 65 feet, a progressive wave would therefore advance at the rate of $3.36\sqrt{65}=27$ nautical miles per

hour. To cover the 65-mile stretch from Cornfield Point Light Vessel to Execution Rocks, a progressive wave would require very nearly 2½ hours.

Examining now the characteristics of the current in Long Island Sound, we find from the current tables published by the Coast and Geodetic Survey that over the greater part of the sound the current is very nearly simultaneous. Furthermore, in relation to the tide it is found that the strength of the current comes approximately midway between the times of high and low water, while the slack of the current occurs about the times of tide.

In Long Island Sound, therefore, the tidal movement presents none of the characteristics of progressive-wave motion. But if we examine the stationary-wave movement in half the tank of water represented in Figure 30, say from C to AD, it becomes evident at once that the tidal movement in Long Island Sound is of the stationary-wave type, exemplified by the half of a simple stationary wave in a tank of water. We shall find later, in discussing the tidal and current phenomena in the Bay of Fundy, in which body of water the greatest known rise and fall of the tide occurs, that here likewise the tidal movement is of the stationary-wave type. On the Pacific coast, Cook Inlet in Alaska may be cited as a body of water in which the tidal movement is similar to that in Long Island Sound.

How are we to explain the fact that of coastal and inland bodies of water deriving their tides from the open sea some sustain progressive-wave motion, while others sustain stationary-wave motion? Are we to look for the answer in differences in the characteristics of the tides that sweep into these bodies of water, or is the answer to be found in the geographic features of the bodies of water? The investigations of the mathematician have shown that the answer is to be found not in any peculiar features of

the tide that enters a waterway, but in the configuration of the waterway, or more precisely in the length and depth of the body of water.

It can be shown from theory, and it has also been verified by experiment, that a body of water opening at one end into a larger basin can sustain a stationary-wave movement of the character exemplified by the movement of the water in half the tank represented in Figure 30. Furthermore, each such body of water has a natural period of oscillation which depends on the length and depth of the body, the formula for a rectangular tank being approximately $T=\frac{4L}{\sqrt{gh}}$ where T is the period of oscillation, L the length of the body in the direction of oscillation, g and h, as before, being the acceleration of gravity and the depth of the water respectively.

Now if a force, capable of sustaining wave motion, acts periodically on a body of water opening at one end into a larger basin, either stationary- or progressive-wave motion will result, depending on the relation of the period of the force to the natural period of oscillation of the body of water. If the two periods are approximately equal, stationary-wave movement will result, while if the two periods differ much, progressive-wave movement will result. Knowing the length and depth of a tidal body of water, we can, by the aid of our last formula, determine in advance of observations whether the tidal movement is of the progressive or stationary-wave type. Taking Long Island Sound with a length of 80 nautical miles and an average depth of 65 feet, we find by substituting in the formula, and remembering that a nautical mile has a length of 6,080 feet and that g has a value, on the average, of 32.17 feet per second, $T=\frac{4\times 80\times 6080}{\sqrt{32.17\times 65}}=42{,}550$. Since the units used in the formula are feet and seconds, the value of T derived above is in seconds, and expressing it in hours we get 11.8 hours.

The natural period of oscillation of Long Island Sound is thus found to be approximately 12 hours. It is therefore to be expected that the periodic force of the ocean tide which sweeps into Long Island Sound, and which likewise has a period of approximately 12 hours, will set up and sustain in the Sound a stationary wave. And it is in the stationary-wave movement that we find the explanation of the characteristic features of the tides and currents in this body of water.

One other feature in the relationship of current to tide must be considered, and that is the relation of range of tide to strength of current. We have found previously that at any given point the velocity of the current varies approximately as the range of the tide. As a local rule, therefore, it would be correct to say "big tides, strong currents; small tides, weak currents." But only as a local rule is this correct, for it does not at all follow that of two places having different ranges of tide, the larger range is accompanied by the stronger current. Many examples might be cited—Long Island Sound itself furnishes a good one. In the Race, the ocean entrance to Long Island Sound, the tide has a range of 2½ feet, while the velocity of the current at strength is 3½ knots. At Execution Rocks, near the western end of the Sound, the range of the tide is nearly 7½ feet, while the velocity of the current at strength is but half a knot.

Two factors determine the velocity of the current in coastal or inland tidal waters at any point along the channel. The first is the volume of water that must flow through the channel in a tidal period past the point, and the second is the area of the cross section of the channel at that point. It is the latter factor that generally makes the current, more than the tide, immediately responsive to changes in geographic or hydrographic features. A change in the cross-sectional area of a tidal waterway produces

a change in the velocity of the current, even though the range of the tide be not perceptibly changed.

In a channel of uniform width and depth the effect of friction on a progressive tide wave is to decrease the range of the tide and the velocity of the current from the mouth upward. In a stationary tide wave, however, the range of the tide increases from mouth to head, but the velocity of the current must decrease, since the volume of water passing any given cross section decreases from mouth to head.

Strong currents are frequently found to occur in short straits connecting two independent tidal waterways. Thus, the strongest currents in New York Harbor are found in East River, which really is no river at all, but a strait connecting Upper Bay with Long Island Sound. Seymour Narrows in British Columbia and Sergius Narrows in Alaska are examples of such straits on the Pacific Coast and in these waterways strong currents likewise occur—5 to 7 knots in Sergius Narrows and 8 to 10 knots in Seymour Narrows. These currents, however, are strictly not tidal currents but are due to differences of level in the two bodies of water which such straits connect. This matter will receive further consideration in the discussion of the tidal movement in the East River in the chapter devoted to flood and ebb in New York Harbor. And it will be found that in this type of tidal movement, which we may call the hydraulic type, the relation of current to tide differs from that obtaining either in progressive-wave or stationary-wave movement.

In conclusion it may be observed that in any given body of water the tidal movement does not conform with absolute exactness to the simple mathematical movements of theory. Existing bodies of water are not of simple geometrical form, and irregularities in shore line and in depth introduce complicating factors. Furthermore, the

tidal movement in a body of water may consist partly of a progressive wave and partly of a stationary wave. Nevertheless, as the examples discussed in this chapter go to show, the actual tidal movements and the relation of current to tide in coastal and inland bodies of water find very satisfactory explanation in the general notions of progressive-wave and stationary-wave movements.

CHAPTER IX

OFFSHORE TIDAL CURRENTS

THUS far in our consideration of currents we have kept close to the coast, discussing only the currents in coastal or inland bodies of water. Suppose now we venture farther away from the coast and see what phenomena the currents in the open ocean present.

Obviously it is a much more difficult matter to obtain systematic current observations in the deep waters of the open sea than in the shallower coastal or inland waters. Barges or small boats will no longer prove suitable; a staunch vessel will be necessary. And the anchoring of a vessel in deep waters presents many difficulties. Furthermore, the cost of maintaining a good-sized vessel with its necessary complement of men is considerable, and it is to be noted that, since the currents in the open sea are not as strong as in the restricted channels of inland waters, the effects of wind and wave and weather are more pronounced, so that the regularity of the tidal currents is frequently masked, and a longer series of observations are generally necessary to disentangle the periodic tidal current from the effects of other agencies.

Fortunately for our purposes, however, we may make use of the lightships maintained by our government along the coast as aids to navigation. Suppose we begin our observations on offshore tidal currents on Nantucket Shoals Light Vessel off the coast of Massachusetts, anchored in a depth of about 180 feet. This light vessel is stationed farthest from the coast, about forty miles southeasterly from Nantucket Island, and is the first aid to navigation

picked up by the ocean liners heading for New York from across the Atlantic.

September 24, 1919, was a pleasant day, and we may therefore imagine ourselves on board Nantucket Shoals Light Vessel on that day, prepared to take observations on the velocity and direction of the current at the beginning of each hour for the whole of the afternoon. For the present, we are interested only in the current near the surface and for this purpose a current pole and log line will be quite satisfactory. Promptly at noon we begin our observations, making use of the ship's compass for determining the direction of the current and recording the velocity and direction in a record book. And since the wind may help in explaining any unusual features of the current we also estimate the velocity and determine the direction of the wind and record these, too, in our record book. The table following gives the results of our observations, the velocity of the current being given in knots, while that of the wind is in miles per hour.

CURRENT OBSERVATIONS, NANTUCKET SHOALS LIGHT VESSEL, SEPTEMBER 24, 1919

Time P.M	CURRENT		WIND	
	Velocity	Direction	Velocity	Direction
Noon	1.1	N79° E	8	N
1	1.0	S 45° E	8	"
2	1.0	S 2° E	8	"
3	1.1	S 32° W	8	NE
4	1.3	S 43° W	8	"
5	1.3	S 55° W	8	"
6	1.2	S 77° W		"
7	0.8	N47° W	calm	.....
8	1.0	N24° W	calm	.....
9	1.0	N13° W	calm	.....
10	1.2	N 2° W	calm	.....
11	1.2	N43° E	calm	.....
midnight	1.0	N63° E	8	SW

A glance at the table shows that the behavior of the current at Nantucket Shoals Light Vessel is of an entirely different character from that in rivers. In the first place, the velocity of the current does not show the rhythmic variation from slack to strength and slack again. Throughout the 12-hour period of our record the velocity of the current was approximately constant. Furthermore, instead of flowing in the same general direction for a period of about 6 hours, and in the opposite direction for the succeeding period of 6 hours, the direction of the current here shows a progressive change at each hourly observation.

It will aid us in visualizing the characteristics of the current at Nantucket Shoals Light Vessel if we plot the results. But it is to be noted that we cannot adopt the same scheme here as in the case of river currents, for we want to represent the constantly changing direction of the current. We must therefore plot the observations in the manner shown in Figure 31. Here each line represents the velocity of the current at the beginning of each hour by its length measured from the center, and the direction of the current by its direction.

The wind during the period of our observations, as shown by the tabular values above, did not attain the velocity even of a gentle breeze. The rotary character of the current shown by Figure 31 cannot therefore be ascribed to the effects of wind and weather. Further observations would only confirm as thoroughly characteristic of the current at Nantucket Shoals the features shown in the diagram. No period of slack occurs here, the tidal current at all times running with a velocity of approximately one knot. And the current shows a progressive change in direction, rotating in the direction of the hands of a clock at an average rate of approximately 29° per hour, so that in a period of $12^h\ 25^m$, it will have veered completely round the compass.

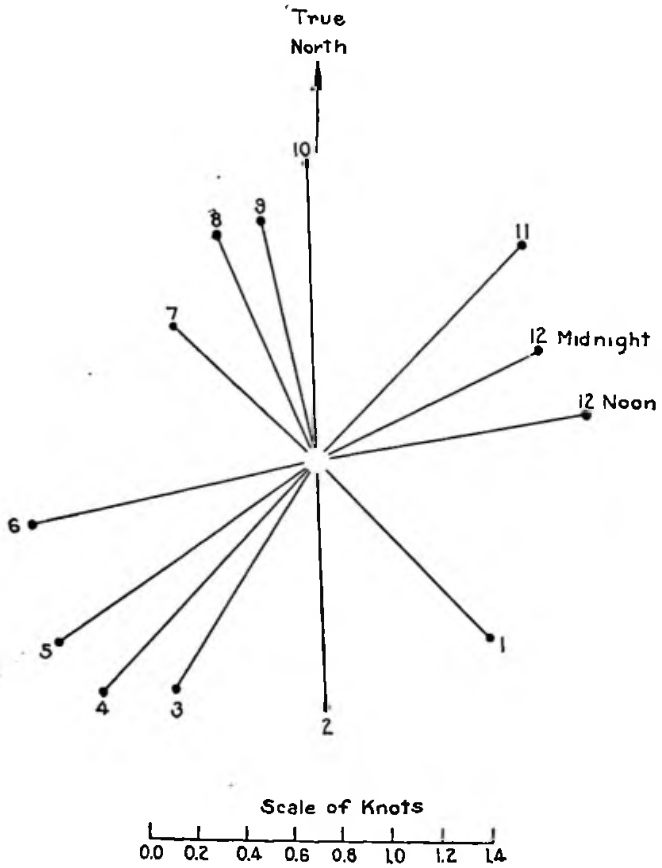


FIG. 31.—CURRENT CURVE, NANTUCKET SHOALS LIGHT VESSEL, AFTERNOON OF SEPTEMBER 24, 1919

Individual observations are subject to inaccuracies of a greater or lesser magnitude, especially in current observations, since these observations are made from a vessel which frequently rolls and pitches on the disturbed surface of the

sea. To eliminate such accidental errors, it is customary to take the average of a number of observations. For the year 1920 there are at hand the results of continuous hourly observations on Nantucket Shoals Light Vessel for the whole month of July. But since the tidal day does not coincide with the civil day, we may either refer the hourly velocity and direction of the current to the moon's transit or, what amounts to the same thing, to the times of high and low water at some near-by port. In Figure 32 the current curve for Nantucket Shoals Light Vessel for July, 1920, is shown with reference to the times of tide at Boston, Massachusetts, H standing for the time of high water and L for the time of low water.

The hourly current lines in the diagram of Figure 32 show a decided regularity; indeed, it is a fairly regular ellipse that is seen to result from joining the ends of these lines. The diagram shows that about 2½ hours before high water at Boston the current at Nantucket Shoals has a maximum velocity in a northeasterly direction. For a little more than 3 hours thereafter the velocity of the current decreases, so that about one hour after high water at Boston the current has a minimum velocity in a southeasterly direction. In the following period of somewhat more than 3 hours, the velocity of the current increases so that about 2½ hours before low water the current has a maximum velocity in a southwesterly direction. Another period of about 3 hours of decreasing velocity now follows, culminating in a minimum velocity a little after the time of low water at Boston, and the cycle is then completed by a 3-hour period of increasing velocity.

To distinguish the type of tidal current found at Nantucket Shoals Light Vessel from that occurring in the relatively narrow channels of rivers and bays, the former is classed as a rotary current, while the latter is called a reversing or rectilinear current. The characteristic features

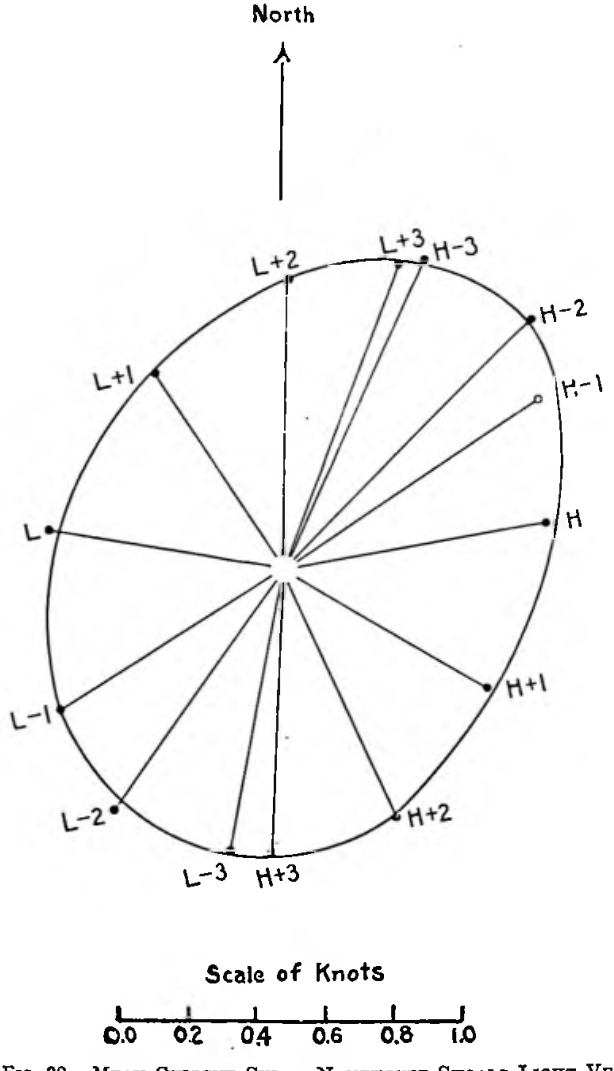


FIG. 32.—MEAN CURRENT CURVE, NANTUCKET SHOALS LIGHT VESSEL

of rectilinear currents are distinct periods of flood and ebb during which the direction of the current is constant, the change from flood to ebb being marked by slack water. The characteristic feature of the rotary current is a constantly changing direction without any slack water.

The currents shown in Figures 31 and 32 represent the current phenomena near the surface. How does the current at some distance below the surface behave? Observations made on Nantucket Shoals Light Vessel at a depth of 100 feet from the surface show exactly the same characteristics as the current near the surface. And this was to be expected, for it is to be recalled that in tidal flow the motion extends from the surface to the bottom; that is, the velocity and direction of the tidal current (except in very shallow waters) from surface to near the bottom are practically the same.

In so far as moderate depths are concerned, the above statement finds confirmation in observations made in various parts of the world. And while actual measurements of current velocities and directions have not been made in depths comparable to ocean depths, there can be no reasonable doubt as to the applicability of the above statement to deep waters. It is based on the known laws which govern the motion of liquids and is confirmed by many facts connected with the configuration of the ocean bottom which find explanation only on the assumption of currents near the bottom similar to those near the surface. It is to be noted, however, that the above remarks refer only to the tidal current. Currents brought about by the wind behave in an entirely different manner and are strongest near the surface. It may therefore happen that currents actually measured near the surface and near the bottom will show apparent differences because of the effect of wind currents. This matter we will consider further in discussing the effects of wind and weather.

In general, it may be said, that offshore currents exhibit the characteristics of rotary currents. Observations made in various parts of the world bear this out. But it is to be observed that such currents reveal differences in two features. In some places the direction in which the current rotates is contrary to that of the hands of a clock. And the relation between the minor and major axes of the current ellipses may vary widely from place to place; that is, we find all forms of current ellipses, ranging from the nearly circular to the very narrow.

With regard to direction of rotation and form of ellipse the offshore currents on the Atlantic coast of the United States resemble those found at Nantucket Shoals Light Vessel and represented diagrammatically in Figure 32. And that figure, it is to be recalled, represents the average current. Throughout the lunar month this current exhibits variations much like the tides or like the rectilinear current in the Atlantic coast tidal rivers. When the moon is full or new, the ellipse representing the currents is larger than the average. Likewise at times of the moon's quadratures, that is when the moon is in its first or third quarter, the ellipse is smaller than the average. The rotary current, too, shows the impress of the declinational effects of the moon; when the moon is over the equator the two successive ellipses of the tidal day are very much alike, while when the moon is at its extreme declination the two ellipses show considerable differences. Figure 33 shows these variations for the month of July, 1920.

On July 9, 1920, the moon was in its last quarter, and on the 23rd of July it was in its first quarter. The upper left-hand diagram in Figure 33 shows the current as observed on those days, referred to the times of high and low water at Boston. It will be seen that while the direction of the current for the different hours is much like that in Figure 32, the velocities are less. In the upper right-

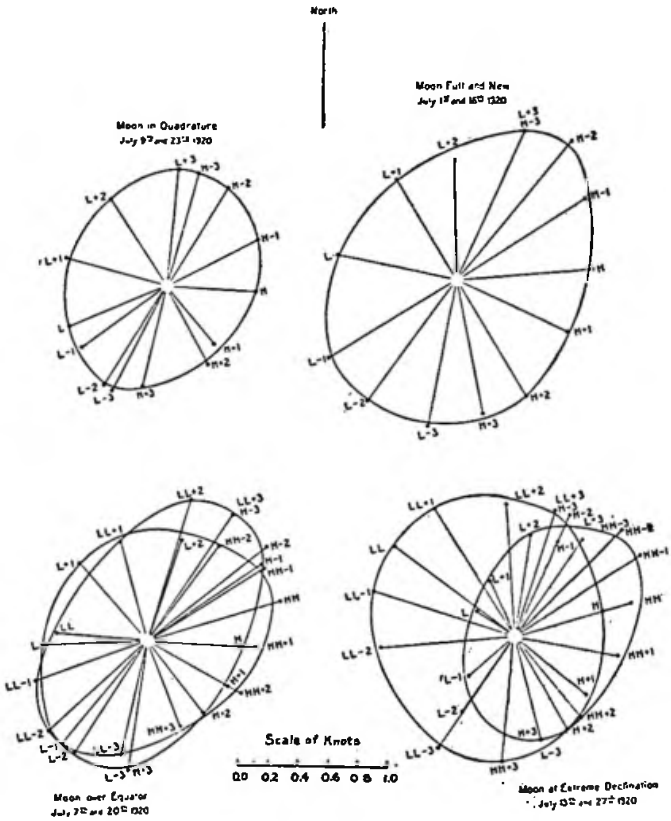

North
Moon in Quadrature
July 9d and 23d 1920
Moon Full and New
July 1st and 16th 1920
Moon over Equator
July 7th and 20th 1920
Scale of Knots
0.0 0.2 0.4 0.6 0.8 1.0
Moon at Extreme Declination
July 3d and 27th 1920

FIG. 33.—CURRENT CURVES, NANTUCKET SHOALS LIGHT VESSEL, FOR VARIOUS PHASES OF THE MOON

hand diagram of Figure 33 is represented the current during the two days of the month when the moon was full and new, when the so-called spring tides occurred. The diagram for those days shows a velocity for the current greater than the average.

In the lower left-hand diagram of Figure 33, the two current ellipses of the tidal day have been plotted separately and represent the current on July 7 and 20 when the moon was over the equator. To distinguish in this diagram between higher high water and lower high water at Boston, the former is designated as HH, while the latter is designated as H. Similarly in this diagram LL represents the time of lower low water, while L stands for the time of higher low water. Broadly, it may be said that the morning and afternoon current ellipses for the days when the moon was over the equator resemble each other closely or, in other words, the current on these days exhibits practically no diurnal inequality. Comparing now the lower right-hand diagram we see a decided difference in the morning and the afternoon current ellipses. This latter diagram represents the velocity and direction of the current as observed on the 13th and 27th of July when the moon was at its greatest north and greatest south declination, respectively, for the month.

The rotary current thus exhibits all the variations with respect to changes in the moon's position found in rectilinear currents. On the Atlantic coast, it is to be recalled, the tides showed but slight diurnal inequality, and the same may be said of the currents both of the rectilinear and rotary types. On the Pacific coast, however, the tide and likewise the rectilinear current exhibit very marked diurnal inequality. We may therefore expect the offshore currents on the Pacific coast to show greater inequality than the offshore currents on the Atlantic coast, and this deduction is borne out by the observations made on the lightships stationed along the Pacific coast of the United States.

In Figure 34 are shown diagrammatically the results of observations made on Swiftsure Bank Light Vessel in the year 1919. This lightship is anchored in 150 feet of water

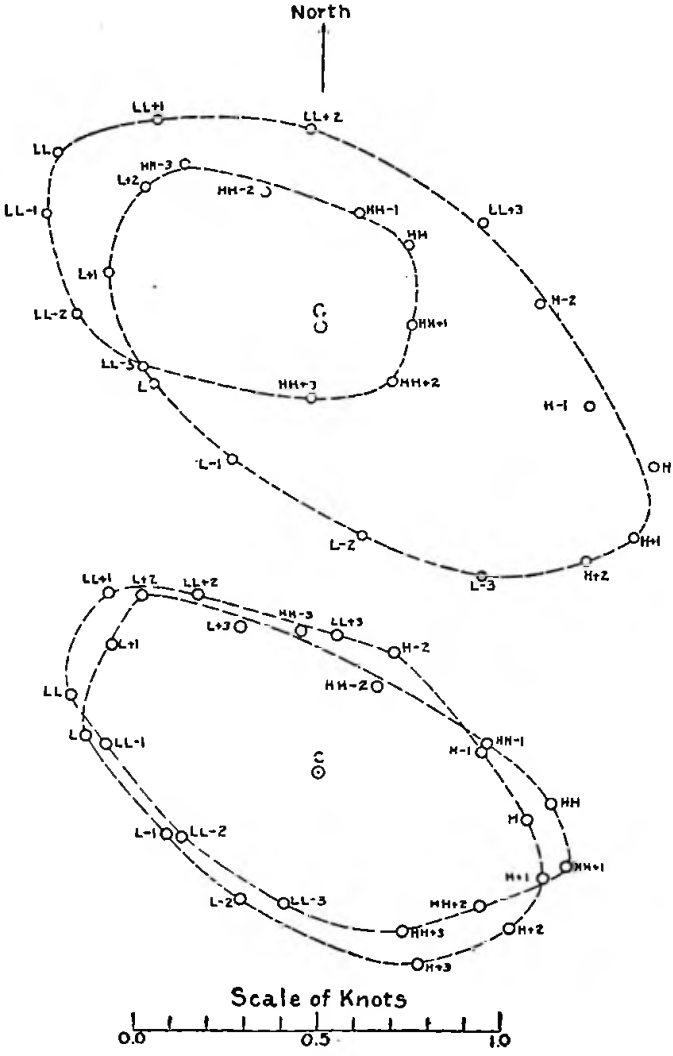


FIG. 34.—TROPIC AND EQUATORIAL CURRENT CURVES, SWIFTSURE BANK LIGHT VESSEL

off the entrance to the Strait of Juan de Fuca, about ten miles from the nearest shore. The velocity and direction of the current are shown for each hour with reference to the times of high and low water at Astoria, Oregon. HH and LL stand, respectively, for higher high and lower low water, and H and L for lower high and higher low water. Lines joining the center of each figure with the various hour marks on the curve give by their lengths and directions the velocities and directions of the current at those hours.

The diagrams show that the offshore current here rotates clockwise as on the Atlantic coast. The upper diagram represents the tropic current, that is, the current on the days when the moon has its extreme semimonthly declination. A comparison of this diagram with the tropic current diagram of Figure 33 shows that at Swiftsure Bank there is considerably greater diurnal inequality than at Nantucket Shoals. The lower diagram of Figure 34 represents the behavior of the current on the days when the moon is over the equator, or the equatorial current. At such times it is seen the two current ellipses of the day resemble each other closely.

All along the Pacific coast the offshore tidal current is characterized by greater diurnal inequality than found along the Atlantic coast. Off San Francisco Bay, the current exhibits even more diurnal inequality than at Swiftsure Bank. Figure 35 shows the mean tidal current curve derived from observations made on board San Francisco Light Vessel, which is anchored in a little more than 100 feet of water about 10 miles southwesterly from the entrance to San Francisco Bay. In this diagram the current is referred to the times of high and low water at San Francisco, HH and LL representing the times of higher high and lower low water, while LH and HL represent the times of lower high and higher low water, respectively.

It is the average condition of the current at San Francisco Light Vessel throughout the lunar month that Figure 35 represents. The very marked difference in size of the

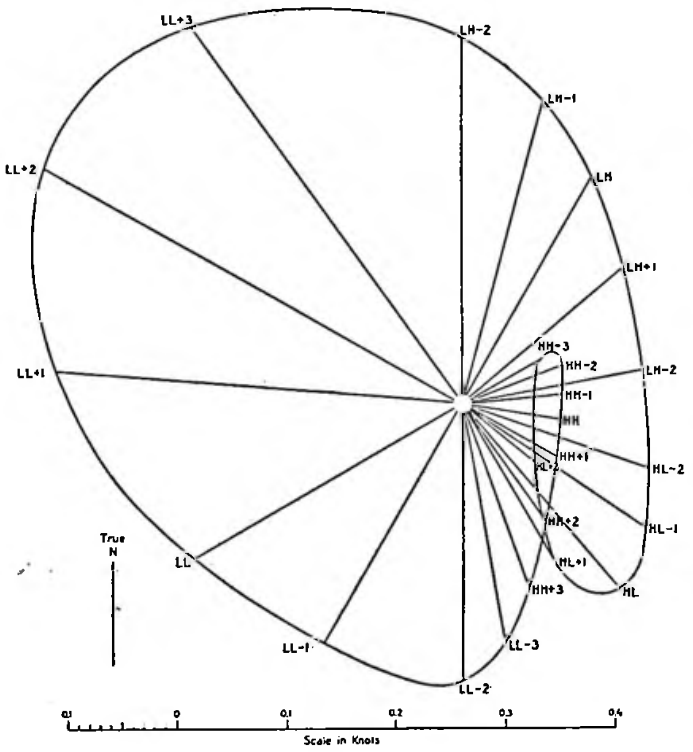


FIG. 35.—MEAN CURRENT CURVE, SAN FRANCISCO LIGHT VESSEL

two current ellipses in the curve of the mean current therefore betokens a very nearly complete elimination of the smaller ellipse at times of the moon's extreme semimonthly declination. And the observations made on this light

vessel bear this out, for at such times the smaller ellipse practically vanishes. When the moon is over the equator, however, the two ellipses of the tidal day are very nearly alike.

A detailed study of the local peculiarities of the offshore tidal currents would lead us too far afield, and we shall, therefore, not pursue this phase of the subject further. We want, however, to discuss briefly the mode of origin of rotary currents and their relation to the rectilinear currents.

In a river channel with its restricted width, it is obvious that the direction of the tidal current must be in the direction of the channel. In other words, in narrow channels the horizontal component of the tidal movement is restricted to a forward and backward movement along a single direction. But offshore there is no longer this restriction in direction. What features would the tidal current exhibit if there were two tidal movements, one along the coast and the other making an angle with the coast? Mathematically, it can be easily shown that the resultant current will be a rotary current, while the resultant rise and fall of the tide would still be a curve approximating the cosine curve.

We may therefore conclude that the rotary character of offshore tidal currents arises from the interaction of tidal movements acting in different directions. These tidal currents will therefore partake of the tidal characteristics of their regions. On the Atlantic coast the tides exhibit little diurnal inequality, and we may therefore expect the rotary current here to show little diurnal inequality; on the Pacific coast the tides are characterized by considerable diurnal inequality and hence the offshore current should also feature this inequality in greater degree than the currents on the Atlantic coast. And this is found to be the case.

As regards its relation to the rectilinear current, the rotary current may be considered as the resultant of two rectilinear currents which cross each other. The velocities and the angle of crossing, as well as the times of strength of these constituent rectilinear currents, may have various relations; and it is these relations that determine the shape and direction of rotation of the current ellipse. Since these relations vary in different regions, the shape of the current ellipse and the direction of rotation also vary.

CHAPTER X

THE MAKING OF THE OCEAN TIDE

EVERY now and then a "new theory" of the tides is announced by some one who tells us that Newton and the other great minds who have labored in this field are all wrong as to the cause of the tide. Some of these "new theories" admit a connection between moon and tide, but make use of a "newly discovered universal principle"; others are less conservative and boldly announce that the moon has nothing to do with the tide. And it is a curious fact that these "new theories," which expound the "true" cause of the tide generally originate with people who live far inland, and not with mariners or others who are familiar with the various phenomena of the tides.

Now it is just as difficult to disprove these "new theories" of the tide when they come up, as it is to disprove the theory that "the moon is made of green cheese." There is no need, however, of wasting any time on such "theories" since they neither help us in understanding the various phenomena of the tides as we find them in nature, nor do they help us in predicting the time and height of the tide for any future time. And these two tests, by the way, are the ones by which the merits of any tidal theory may be readily tested. Unless it helps us in the one or other respect, it is worthless.

That the moon and sun are the causes of the tide had been surmised for many centuries, but it was only with Newton's announcement of the law of universal gravitation,

in the latter part of the Seventeenth Century, that this could be proven. Since that time no one familiar with the tides has questioned that the moon and sun are the causes. Nor is there any question as to the tide-producing forces brought about by the attraction of moon and sun, for these are well known and can be readily determined from known astronomic data. But taking these forces for granted, what is the precise nature of the movements of the ocean waters brought about by these forces?

On the basis of gravitation the mathematician can prove that if the earth were entirely covered with a layer of water of considerable depth, high water would occur with the moon's meridian passage, both upper and lower, and low water about six hours later. Under these conditions the tide might be pictured as a wave moving westward with its crests directly under the moon. And even before the formulation of the law of gravitation, the Ptolemaic system of astronomy with its assumption of a stationary earth and westward revolving heavenly bodies had led to the notion of a westward movement of the tide around the earth. But the continents and islands rising out of the oceans divide the waters of the earth into a number of irregular-shaped bodies having varying depths. This must modify profoundly any theories based on assumptions of uniformity. In fact, the theories that have been advanced to explain the making of the ocean tide are based on a synthesis of the results of tidal observations made in the various oceans.

In 1833 William Whewell presented before the Royal Society of London a paper entitled "Essay Towards a First Approximation to a Map of Cotidal Lines." Accompanying this paper was a map of the world on which were drawn so-called cotidal lines, that is, lines joining points at which high water occurs at the same hour. Parenthetically it may be added that Whewell contributed considerably to

the advancement of tidal knowledge, principally through the analysis and coördination of large masses of tidal data. It was at his suggestion that, in 1835, the United States and several European countries coöperated in securing at a number of places simultaneous observations covering a period of several weeks.

If we glance at a map of the world, we will notice that the only body of water entirely encircling the earth is the Southern Ocean, the belt formed by the waters of the Atlantic, Pacific, and Indian Oceans southward of the great land masses, toward the Antarctic. On Whewell's cotidal map the tide is shown as progressing from south to north in the Atlantic, Pacific, and Indian Oceans, while in the Southern Ocean the tide is shown as progressing westward.

It is to be remembered that tidal observations have been confined almost without exception to the immediate vicinity of the coast, and that over the wide expanses of the open ocean the time of high water is not known from direct observations. The joining by a cotidal line of two points separated by a wide expanse of water can only be made in accordance with certain assumptions, and the entire character of a cotidal map depends on these assumptions. Whewell expressly emphasized this by stating in his conclusion to the memoir: "I shall be neither surprised nor mortified if the lines which I have drawn shall turn out to be in many instances widely erroneous: I offer them only as the simplest mode which I can now discover of grouping the facts which we possess."

Underlying Whewell's cotidal map is the theory of the tide known as the "progressive-wave" theory. This theory considers the tides of the world as due principally to the action of the tidal forces on the broad and deep waters of the Southern Ocean. Here, it is argued, the tidal forces have almost uninterrupted sway; hence there is found in

this region the most favorable conditions for the generation of tidal waves.

In the Southern Ocean, then, the progressive-wave theory makes the tide originate. Here, according to this theory, the tidal forces raise two tidal waves, 180° apart in longitude, which travel this belt of water from east to west, constrained by the moon to keep step with its own motion. As each of these waves sweeps past the Cape of Good Hope, it generates a wave which travels up the Atlantic Ocean, and it is this secondary wave generated by the primary tidal wave of the Southern Ocean which determines the tides on the shores bordering the Atlantic Ocean.

We must note carefully the distinction between the primary tidal wave and the secondary wave which, according to this theory, sweeps up the Atlantic Ocean. The secondary wave travels freely, according to depth. But the primary tidal wave in the Southern Ocean keeps step with the moon; it travels not as a "free" wave, but as a "forced" wave, compelled by the moon to keep in time with its own movement.

Obviously, under the progressive-wave theory, the tide-producing forces acting on the waters of the Atlantic Ocean must be assumed as tending to give rise to a primary tidal wave, such as described for the Southern Ocean. However, the obstruction of land masses to the east and to the west do not permit this tidal wave to become well-developed and therefore the tides in the Atlantic Ocean are considered to be due principally to the secondary free wave brought about by the tidal wave of the Southern Ocean.

If we follow the free secondary wave as it travels up the Atlantic, it is evident that the time of tide should be later as we go northward. And for the continents of South America and Africa, this is actually the case. Observa-

tions show that high water and low water occur later as we go northward along the Atlantic coasts of Africa and South America. From observations, too, we find that the range of the tide becomes greater as we go from Cape Horn to the Amazon. This fact the progressive-wave theory explains as being caused by the narrowing of the ocean channel between northern Africa and Brazil. As the channel narrows, the energy of a wave becomes concentrated, so that the height of the wave, and hence the range, is increased.

The tides in the other oceans are explained by the progressive-wave theory, in a similar manner, as due to the generation of free secondary waves brought about by the primary tidal wave of the Southern Ocean, which keeps rhythm with the movement of the moon. The differences in the times, ranges, and types of the tide are accounted for as being due to differences in depth and width of channel, to differences in the configuration of coast lines and to interferences of tide waves coming from different directions. But as dominating the tides of the world the progressive-wave theory sets up the primary tidal wave that has its origin in the Southern Ocean.

Now it is obvious that the progressive-wave theory has many things in its favor. It is very plausible and easily explains, as we found, certain features of the tide as they exist in nature. Under this theory, too, the "age of the tide" finds ready explanation. It will be recalled that at most places the greatest range of the tide does not occur on the days of full or new moon, but a day or more later. In fact, there is generally an interval between a maximum or minimum position of the moon and the appearance of the corresponding feature in the tides, this retard being known as the "age of the tide." The progressive-wave theory assumes the tide in the Southern Ocean to respond immediately to the moon and sun and the age of the tide

at any place in the Atlantic, Pacific, and Indian Oceans is therefore the time taken by the secondary tide wave to travel from its origin to the place where the tide is observed.

Since Whewell's time our knowledge of the local characteristics of the tide has been very greatly increased by the accumulation of accurate tidal observations at many points in all the seas of the world. And the data from these observations do not accord with the simple explanations of the progressive-wave theory. Examining, for example, the distribution of the age of the tide as it manifests itself with regard to the occurrence of spring tides, we find that for the Atlantic Ocean this age is practically the same from Cape Horn in South America to Cape Farewell in Greenland. If the progressive-wave theory were correct, this age should become greater as we go northward from the Southern Ocean. Under the progressive-wave theory, too the relatively small amount of diurnal inequality in the North Atlantic tides is not easily explained except by interferences of secondary tide waves coming from different directions. In fact, the progressive-wave theory leaves too many of the characteristics of the tide in various regions to be explained by differences in channel and configuration of shore line and interferences of secondary tide waves coming from different directions.

There are other objections that may be urged against the progressive-wave theory. Indeed, from general considerations, it may be objected that in so vast a basin as that of the Atlantic or Pacific Ocean, the tides can scarcely be due to a derived wave from the Southern Ocean, especially in view of the considerable ranges of the tide in the middle northern latitudes. Fifty years ago William Ferrel, of the U. S. Coast and Geodetic Survey, after a careful study of the Atlantic Ocean tides, came to the conclusion that "if there were a dike extending from the Cape of Good

Hope to the coast of South America, the tides of the North Atlantic Ocean would most probably be very nearly the same."

A study of the results of the tidal observations made since Whewell's time brings out the fact that the characteristics of the tides in the various oceans differ considerably, but that for any given ocean basin these characteristics are much the same. In other words, instead of constituting a world phenomenon dominated by the forced tide wave of the Southern Ocean, the tide appears to be a local or regional phenomenon. Is it therefore not probable that the tides in the various oceans are brought about by the action of the tide-producing forces on the waters of the oceanic basins, each basin responding in accordance with its geographical make-up? Furthermore, may not the rise and fall of the tide in the open ocean be due to a stationary tide wave rather than to a progressive one?

From time to time during the past century the stationary-wave movement was suggested as the probable explanation of the very considerable rise and fall of the tide at many places on the open coast. And at the beginning of the present century, Dr. R. A. Harris, a tidal mathematician of the U. S. Coast and Geodetic Survey, made the stationary wave the basis of a new theory of the making of the ocean tide. This newer theory, known as the stationary-wave theory, is diametrically opposed to the ideas advanced by the progressive-wave theory. It does away with the conception of a single world phenomenon and substitutes regional oscillating areas as the origin of the dominant tides of the various oceans. But before taking up this newer theory it will be of advantage to digress for a moment to a further consideration of stationary waves.

A good example of stationary-wave motion is furnished by the movement of the water in a rectangular tank, one end of which is raised and then immediately lowered. This

type of wave is illustrated diagrammatically in Figure 30. The whole mass of water moves rhythmically first to one end of the tank and then to the other, so that it is low water over one half the tank at the same instant that it is high water over the other half. In this type of wave motion the rise and fall of the water is greatest at the two ends, decreasing gradually towards the middle where no vertical motion takes place. This line across the middle of the tank about which the water oscillates is known as the nodal line, and the period of oscillation is given by the formula $T = \frac{2L}{\sqrt{gh}}$ where L is the length of the tank in the direction of oscillation, h the depth of water and g the acceleration of gravity. For each body of water, therefore, there is a definite period of oscillation which is determined by the length of the body and the depth of the water. This period is known as the natural period of oscillation.

Suppose now we take two rectangular tanks of water, alike in all respects, place them together end to end, and start stationary-wave oscillations in each of them in such wise that it is high water or low water at the same instant at the contiguous ends. The motion may then be represented diagrammatically by Figure 36. At the instant when the surface of the water is in position A B in the one tank, it will be in position B D in the other; and when in position E C in the one, it will be in position C F in the other. With this type of movement in the tanks, it is obvious that the partition walls between the tanks may be removed and the movement sustained in the whole body of water which is now contained in a single tank twice the length of either of the original tanks. In our enlarged tank we now have a stationary-wave movement with two nodal lines, at M and N, high water occurring in the middle at the same instant that it is low water at the two ends and low water in the middle being coincident with high water at the two ends.

Reasoning of a similar nature will show that a body of water may sustain a stationary-wave movement with three, four, or more nodal lines. But in every case we may consider it as made up of a number of simple stationary-wave movements, each with one nodal line, which then permits us to determine the period of oscillation in terms of length and depth as expressed in our formula above. Furthermore, a body of water not rectangular in shape may also sustain a stationary-wave movement with several nodes. For instance, in our example represented by Figure 36, we might have taken two tanks, the bottoms of which instead of being rectangular were trapezoidal, so that when put together they would form an L-shaped figure. Without

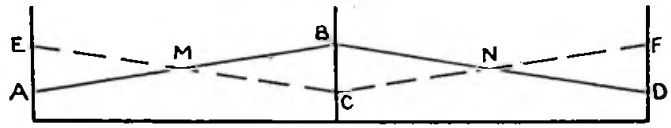


FIG. 36.—BINODAL STATIONARY WAVE

pursuing this phase of the subject further, it is evident that we may have stationary-wave movement in basins of various shapes and with one or more nodes.

In swinging a child we know that a slight force repeated at regular intervals is sufficient to maintain a vigorous swinging. In the same way a slight periodic force will be sufficient to maintain a vigorous stationary-wave movement in a tank or other body of water. However, the question immediately arises as to whether there is any relation between the period of the force and the period of oscillation of the body of water. To determine this we may take a number of tanks of water of varying lengths and start stationary-wave oscillations in them. If now we make some arrangement for applying periodically a slight force to the tanks, it will be found that for any given

period the different tanks respond differently. Some will sustain a vigorous oscillation, others only a moderate oscillation, while some will show very little oscillation. Investigating the matter further, we will find that the motion is greatest in those tanks whose period of oscillation approximates closest the period of the applied force, while the motion is least in those tanks whose periods of oscillation differ most from the period of the force.

Summing up, we may say that each body of water has a natural period of oscillation, and that it responds most to a periodic force when the period of the force approximates the natural period of oscillation. Furthermore, if a number of bodies of water are subjected to several periodic forces acting simultaneously, each body of water will respond with stationary-wave oscillations, the periods of which correspond to the periods of the forces; but in each body of water that oscillation will be best developed which approximates closest to the natural period of the body.

Now to come back to the tides, we have periodic forces in the tide-producing forces of sun and moon. Of these periodic forces, the principal ones are the semidiurnal tide-producing forces with periods of $12^h\ 00^m$ for the solar and $12^h\ 25^m$ for the lunar, and the diurnal tide-producing forces with periods of approximately $24^h\ 00^m$ for the solar and $24^h\ 50^m$ for the lunar. According to the stationary-wave theory these forces set up and maintain in various areas or portions of the oceans stationary waves having periods which correspond with the periods of the tide-producing forces; but in each of the various portions of the oceans the stationary waves best developed are those which have periods approximating the natural period of oscillation of the particular oceanic area in question.

If we take a map of the world which shows the depths of the oceans, we can separate out areas which have the requisite length and depth to support stationary waves

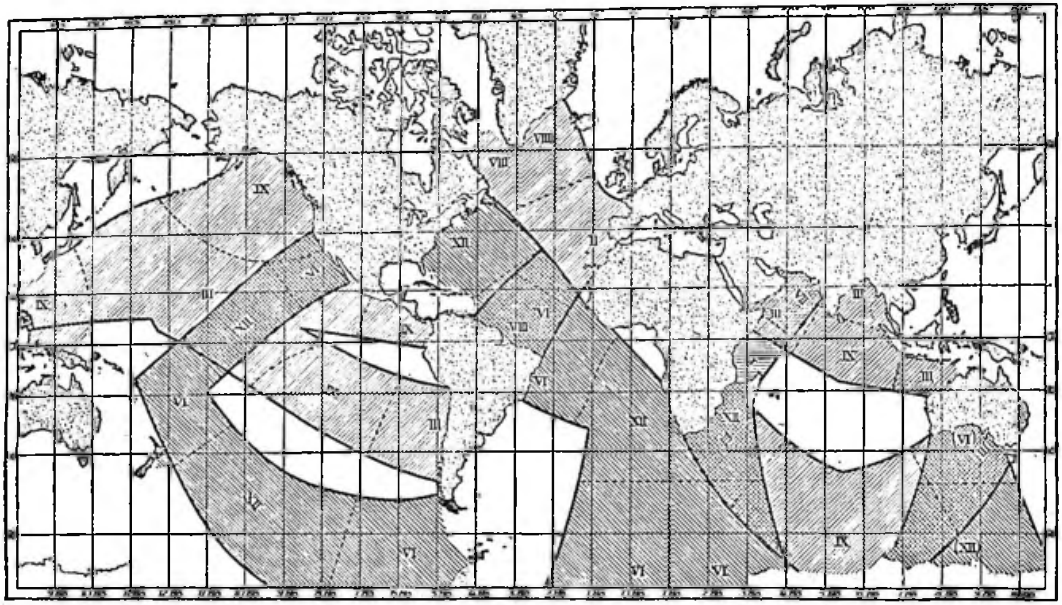


FIG. 37.—OCEANIC OSCILLATING SYSTEMS FOR THE SEMIDAILY TIDE-PRODUCING FORCES, ACCORDING TO HARRIS

corresponding to the periods of the principal tidal forces. Dr. Harris has done this and the oscillating areas or systems that correspond to the semidaily forces of the moon are shown in Figure 37. It is to be recalled that in our tanks of water the width of the tank did not enter into the period of oscillation, hence it is not necessary that the oceanic oscillating areas should have lateral land boundaries to sustain stationary waves. A first glance might lead one to think that the oscillating areas or systems represented in Figure 37 were drawn subject merely to fancy. It may therefore not be out of place to state here that these oscillating systems were drawn in conformity with mathematical and physical principles. And any one in a position to digest solid mathematical food can do no better than to work through Dr. Harris' *Manual of Tides* in which the mathematical development of his tidal theory is outlined.

The Roman numerals on the oscillating systems of Figure 37 give the hours (in lunar time) after the moon's passage over the meridian of Greenwich when high water occurs over the area indicated. Several of the oscillating systems are overlapping and these are distinguished by different kinds of shading. The nodal lines of the various systems are indicated by dashed lines across the shaded areas. The unshaded portions of the oceans are areas of such size and depth that the action of the tide-producing forces on these areas can produce but little tide. In such regions, therefore, the tides that occur are due to the tides produced in the neighboring oscillating areas.

According to the stationary-wave theory of the tides the semidiurnal tide-producing forces of the moon, as they sweep over the oceans, maintain a stationary-wave oscillation in these open ocean systems, the rise and fall being greatest at the ends and least at the nodal lines. These oscillations give rise to the dominant semidaily tides

of the world. Irregularities in depth and openings in the coast line cause the stationary-wave tides generated in these ocean basins to send off progressive waves into other parts of the ocean and into coastal tidal waters.

How well does this newer theory accord with the times and heights of the tide as actually observed? Taking the eastern coast of the United States we see from Figure 37 that this forms one end of an oscillating system in which high water should occur 12 lunar hours after the moon's passage over the meridian of Greenwich. And, as a matter of fact, careful tidal observations on the open coast show that from Halifax in Nova Scotia to Cape Canaveral in Florida, the time of tide is practically the same, occurring about 12 lunar hours after the moon crosses the meridian of Greenwich.

It will be noted that the nodal line of that part of the Atlantic Ocean system that controls the tide on the coast of the United States runs northeast and southwest. As we approach the nodal line of a stationary wave, the rise and fall of the tide should become less and less. In the open ocean, unfortunately, the measurement of the rise and fall of the tide is a problem of very considerable difficulty, so that the only place where observations can readily be made to discover the existence or nonexistence of this nodal line is at its southwest end where it approaches the Windward Islands. Fortunately, we have here islands stretching all along from Florida to the point where the nodal line is shown to strike the Windward Islands. And from observations we find that on the coasts of Georgia and Florida the rise and fall of the tide is about 6 feet; for the Bahamas it is 2 feet; for the outer coast of Porto Rico it is 1 foot and further eastward the rise and fall of the tide is practically nothing. The existence of this nodal line therefore appears established beyond question.

It will be recalled that the tides at the two ends of the

Panama Canal showed striking differences—a range of 12 to 16 feet on the Pacific end and a range of less than a foot on the Atlantic end. Does the stationary-wave theory throw any light on this matter? Glancing at our oscillating systems represented in Figure 37 we find that the Pacific Ocean entrance to the Canal is situated at one end of an oscillating system; the tide should therefore have a considerable rise and fall. On the Atlantic side the Canal communicates with the Caribbean Sea, which, together with the Gulf of Mexico, is cut off from the oscillating systems of the Atlantic Ocean by islands and reefs that stretch from Florida to the northern coast of South America. And it so happens that the body of water made up by the Gulf of Mexico and the Caribbean Sea is, with respect to the semidaily tidal forces, not of the proper length and depth to sustain any considerable semidaily oscillation; but it is of the proper shape to respond to the daily tide-producing forces. Hence we find here a daily tide with a small rise and fall.

Between Australia and the antarctic continent, Figure 37 shows an oscillating area in which the Roman numerals are enclosed in parentheses. This area is of such dimensions that it has a period of oscillation more nearly that of the solar semidaily forces than that of the lunar. Consequently, the tides here should show a relatively large solar constituent. And this is borne out by the analyses of the tidal observations made on the southern coast of Australia.

The stationary-wave theory of the tides explains also very nicely the behavior of the tide on the shores of the island of Tahiti. This island, one of the most romantic of the South Sea Isles, has for many years stood as a symbol of freedom from the restraints of civilization. Here even the tides appear, at first glance, to have thrown off the restraining influence of the laws that govern the tides over the rest of the world; for at Tahiti, the tides do not

follow the moon. Instead of behaving like the tides with which we are familiar along our own coasts, and coming later each day by about 50 minutes, the tides at Tahiti come about the same time every day. With a range of a little less than one foot, high water comes, day after day, about midnight and noon, and low water approximately at 6 in the morning and 6 in the evening. The tides therefore follow the sun and not the moon.

In accordance with the progressive-wave theory, this strange behavior of the tide at Tahiti could be explained as due to the interference of two tide waves coming from different directions, the lunar constituents of which have approximately equal ranges and are 6 hours apart in time; so that high water of the one coincides with the low water of the other. The lunar tide is thus neutralized, and it is only the solar tide that shows any rise and fall. This, to be sure, is a perfectly satisfactory explanation, except that no physical reason can be assigned for the meeting of these tide waves in the South Pacific Ocean near Tahiti rather than at other places.

How does the stationary-wave theory explain the unusual behavior of the tide at Tahiti? If we glance at Dr. Harris' oscillating systems for the lunar semidiurnal forces, we find that Tahiti lies very close to a nodal line. The lunar tide must therefore here be very small. But Tahiti is not close to a nodal line of the solar semidaily oscillating systems. Hence, the tidal rise and fall takes place principally in response to the solar tidal forces. It is to be noted that in this explanation, the stationary-wave theory gives a physical reason for the almost complete vanishing of the lunar tide, for the position of the nodal line is conditioned by the depth, length, and shape of the ocean basin.

With respect to the daily tide-producing forces, Dr. Harris sought in the oceans for areas having the proper

dimensions to sustain stationary waves with a period of a day. He was unable to find such areas in the Atlantic Ocean, but he did find them in the Pacific and Indian Oceans. This, then, explains the existence in the Pacific and Indian Ocean tides of considerable diurnal inequality and the relative insignificance of this feature in the Atlantic Ocean tides. For it is to be recalled that diurnal inequality arises from the combination of a daily tide with a semidaily tide, the greater the daily constituent the greater the inequality. In the Atlantic Ocean the daily tide is not well developed because there exist in this ocean no areas having the proper dimensions for sustaining a well-developed stationary wave with a period of a day. The existence of such areas in the Pacific and Indian Oceans gives rise to a well-developed daily tide and thus to considerable diurnal inequality.

The oscillating systems belong to the open ocean. Where openings occur in the vicinity of an oscillating system, progressive waves are formed which travel up these openings. Shoals likewise give rise to progressive waves. And since shoals and openings occur most frequently near the coast, it is obvious that the tides observed along the coast and in inland tidal waters will exhibit characteristics differing in many respects from the simple character of stationary-wave oscillation. Nevertheless, in the tide on the open coasts of the various oceans there is considerable evidence in favor of this newer theory of the making of the ocean tide that does away with the conception of a single world phenomenon and substitutes regional oscillating areas as the origin of the dominant tides of the various oceans.

CHAPTER XI

EFFECTS OF WIND AND WEATHER

WATER is an extremely mobile substance. The oceans respond therefore not only to the tide-producing forces which are celestial in origin and world-embracing in extent, but also to terrestrial forces of local character. In other words, the regularity of the tidal movement of the waters on the earth is subject to the disturbing influences of wind and weather. And these disturbing influences affect both the tide and the current.

It is a matter of common knowledge that a wind blowing from the sea tends to raise the level of the sea along the coast, while a wind blowing from the land tends to lower it. This obviously is due to the fact that in the one case the wind creates a current which transports water toward the coast, while in the other case the water is carried away from the coast. With landward winds, therefore, high water will rise higher than the average, while the depth to which low water falls is less than the average. Similarly, winds blowing seaward reduce the height of high water and augment the depth to which low water falls.

This effect of the wind, it must be noted, is brought about not by any effect of the wind on the tide-producing forces, but by the effect of the wind on the level of the sea. The amount of tidal rise or fall is changed by the wind only as this rise or fall takes place from a higher or lower level of the sea. Hence with a steady wind from a given direction, after a sufficient time has elapsed to

permit the establishment of an equilibrium between wind and sea level, the range of the tide would, in general, show no change.

Steady winds, however, are not the usual state of affairs; and in consequence of the variability of the wind, the range of tide is frequently modified to a considerable extent. To see how this is brought about, let us assume, for the sake of simplicity, a strong wind to begin blowing from the sea at the time of low water and to continue throughout the rise of tide; then promptly at high water assume it to change direction and begin blowing toward the sea with the same strength. During the rise of the tide the wind has been transporting water toward the coast and hence the height to which high water rises will be greater than normal, that is, the range will be increased. Similarly, during the succeeding fall the wind will be transporting water from the shore and therefore low water will fall lower than would otherwise be the case, and again the effect of the wind would be to increase the range of the tide.

If we reverse the conditions in the previous paragraph and assume the wind to be blowing seaward during the rise and landward during the fall, a moment's consideration will show that the range of the tide would be decreased in each case. With respect to the range of the tide, therefore, the wind may either increase it, decrease it, or leave it unchanged, depending on the direction and duration of the wind. But it is to be noted that the effect of the wind on the range of tide is not everywhere the same, for it is very greatly modified by such local features as the extent of exposure to wind, configuration of shore line, and depth of water. It is obvious that the wind has its greatest influence on the surface of a body of water, this influence decreasing rapidly from the surface downward. Other things being equal a wind of given strength

will raise or lower the level of a shallow body of water to a greater extent than that of a deep body of water.

As regards the time of tide it may be said that, in general, steadily blowing winds do not disturb the times of high and low water much, or in other words, the form of the tide curve is not distorted. With variable winds, however, especially during heavy storms, the times of high

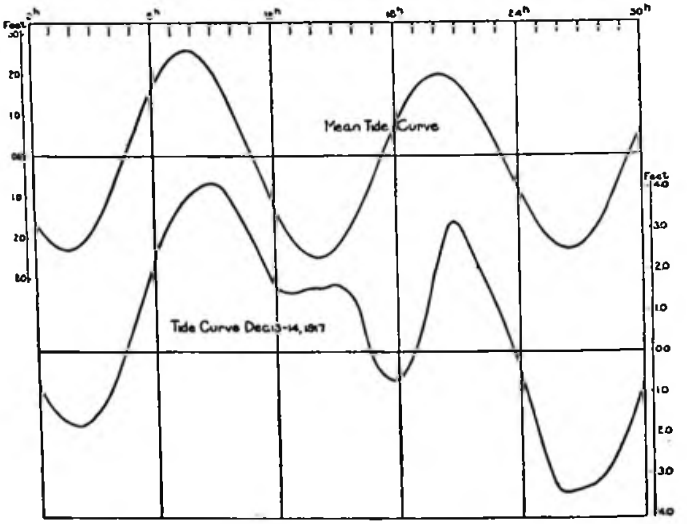


FIG. 38.—MEAN TIDE CURVE AND STORM TIDE CURVE, FORT HAMILTON, NEW YORK

and low water and the form of the tide curve may be considerably changed. An example of this is seen in Figure 38, the upper curve of which represents the mean rise and fall of the tide at Fort Hamilton in New York Harbor, while the lower curve represents the rise and fall of the tide at Fort Hamilton for December 13 to 14, 1917, on which days heavy northeast to northwest winds prevailed.

To make the tide curves comparable in time, they are referred to the time of the moon's transit across the meridian of Fort Hamilton, zero hours being the instant of the moon's upper meridian passage. For the lower curve zero hour corresponds to 11:10 A.M. on December 13, 1917.

A comparison of the two curves shows that for the first 6 hours the two tide curves agree closely; but from that time on the increasing velocity of the wind on December 13, 1917, is reflected in a disturbed condition of the tide, the high water being retarded by an hour and the following low water by 4 hours. This very considerable disturbance in the time of low water was evidently brought about by a rapid rise of sea level between 12^h and 15^h (11:00 P.M. December 13 and 2:00 A.M. December 14, 1917). The weather record shows heavy northwesterly winds for the early hours of December 14—between 2:00 and 4:00 A.M., a wind of 88 miles per hour being recorded. With less violent winds for the remainder of the day, the following high and low waters are seen to be much less disturbed in time.

It will be noted, too, that in disturbing the time of high and low water on December 13 and 14 the wind brought about a marked change in the durations of rise and fall of tide. On the average, the tide at Fort Hamilton rises for a period of 6^h 02^m, while the fall occupies 6^h 23^m. For the 30-hour interval represented by the lower curve of Figure 38, we find that the first rise occupied a period of 6^h 55^m, being followed by a fall that lasted for more than 9 hours. The next rise was very short, requiring but 2^h 55^m, while the succeeding fall was completed in 4^h 20^m.

Making use of the weather records and the data derived from automatic tide gauges, it is possible to work out, for any given port, a table which will show the change in the time and height of high or low water due to winds of various strengths and directions. But in doing this we would

find that another element of the weather was making itself felt, and that is the barometric pressure. It is easy to see that this should be so, for if we assume the atmospheric pressure over the sea as constant, the water in any given harbor may be regarded as constituting a huge inverted water barometer. When the barometric pressure in the harbor rises the water will be depressed, while with a falling barometer the water will rise. Since mercury is about thirteen times as heavy as water, it follows from the above that a change in barometric pressure of one inch should be accompanied by a change in the level of the water of a little more than one foot.

As a matter of fact, the barometric pressure over the sea is not constant; hence, the level of the water in a harbor does not vary simply as the local barometric pressure, but rather as the pressure gradient, which defines the relation of the barometric pressure in the harbor to that of the open sea in its vicinity. In connection with the determination of mean sea level considerable study has been given to the problem of the exact relationship between changes in barometric pressure and sea level, but here we must content ourselves with the statement that, in general, it has been found that the water in a harbor behaves approximately as an inverted water barometer, falling when the barometer rises and rising when the barometer falls.

An example of the response of the water level to sudden changes in barometric pressure is shown in Figure 39. The upper curve is a reproduction on a reduced scale of the tide curve made by an automatic tide gauge in San Francisco Bay, just inside the Golden Gate, for the first 16 hours of November 21, 1910, while the lower curve gives the barometric pressure at San Francisco for the same time. The figures along the tide curve give the hours of the day, the horizontal lines near the figures indicating

on the curve the beginning of each hour. The vertical scale is shown to the left, the upper one in feet for the tide and the lower in inches for the barometric pressure.

Glancing first at the tide curve, it will be noted that up to about 4:45 A.M., the curve is quite regular, the small "saw teeth" denoting the rise and fall of the ocean swell coming in through the Golden Gate. At 4:45, after the tide has been falling for about half an hour, there is a

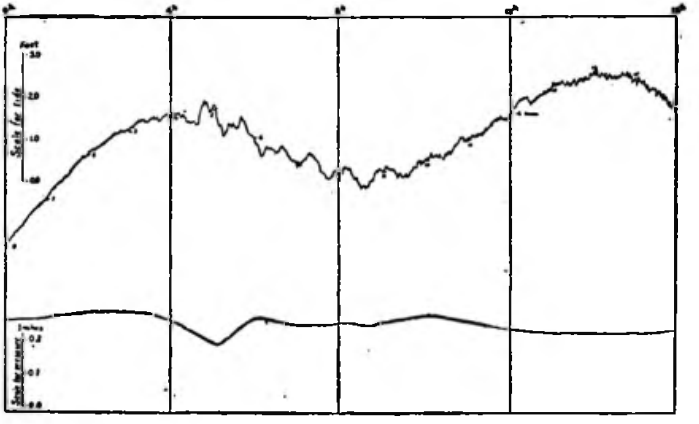


FIG. 39.—TIDE CURVE AND BAROMETRIC PRESSURE CURVE, SAN FRANCISCO, NOVEMBER 21, 1910

sudden rise of more than half a foot, and from this time on for about 6 hours, the tide curve discloses a disturbed condition of the water in the Bay, the regular rise and fall of the tide showing superimposed upon it a nontidal fluctuation with a range of a little over half a foot and a period of somewhat less than an hour.

Now it is to be noted that the disturbed condition of the tide in San Francisco Bay shown in Figure 39 can in no wise be ascribed to the wind, for the weather record shows the wind on that day to have had an average velocity of but

5 miles per hour and a maximum velocity of but little more than 10 miles an hour—technically not even a light breeze. But if we glance at the curve of barometric pressure in Figure 39, a striking fluctuation is seen to have occurred between 4:00 and 6:00 A.M.—a rapid fall of a tenth of an inch from 4:00 to 5:00 A.M. and an equally rapid rise of the same amount from 5:00 to 6:00 A.M. The fluctuation of the water during the same period is seen to agree with that of the barometric pressure—rising during the fall of the barometer and falling during the rise of the barometer. An interesting feature to be noted is that the water in the Bay continues to fluctuate with the same period, but with decreasing amplitude for a number of hours following the return of the barometric pressure to normal conditions. We shall return to a consideration of this fluctuation a little later.

With strong landward winds and a rapidly falling barometer, it is to be expected that a considerable rise in the level of the sea will take place, especially in coastal regions bordering on shallow waters. It is in this combination of circumstances that we have the explanation of the so-called tidal waves which have at times inundated the low-lying regions of the coast fronting the Gulf of Mexico.

The tidal wave, so-called, is, in general, brought about either by severe storms or by earthquakes. Except in name, it has nothing in common with the tide; it is in no way due to the tide-producing forces, nor does it possess the periodicity characteristic of the tidal movement. The appellation "tidal wave" is therefore a misnomer, but so firmly has it become established in the English language that generally no other term is used to designate the destructive waves of the sea that at times overwhelm coastal regions.

The "tidal wave" that results from an earthquake is a striking phenomenon, but one that obviously falls outside

the scope of our subject; but that due to severe storms is merely an exceptional rise of sea level brought about by wind and weather. As an example of this we may consider briefly the "tidal wave" that brought destruction to the city of Galveston, Texas, on September 8, 1900.

Galveston is situated on an island of sand, 30 miles long and from 1 to 3 miles wide, washed up by the waves of the Gulf of Mexico. On the eastern end of this sandy island, the elevation of which in many places was but little above mean high water, the indomitable spirit of a pioneer people reared a modern city and behind its scanty shelter developed one of the world's great ports. This region of the Gulf coast is subject to tropical storms of great intensity; and again and again the sea launched angry waves through the streets of the city as if in warning against further encroachment of a domain it claimed for its own. But each time the challenge was flung back and a greater city arose in defiance of storm and wave.

In the early days of September, 1900, a West Indian hurricane headed toward the Texas coast. At Galveston the barometer commenced falling during the afternoon of the 6th and continued falling slowly up to noon of the 8th, when it registered 29.42 inches. Now it began falling rapidly and at 8:30 P.M. it read 28.48 inches—a fall in pressure of approximately 1 inch in 8½ hours. A heavy swell made its appearance in the Gulf on the afternoon of the 7th, and the water rose until it overflowed the lower parts of the city. The wind did not attain storm velocity till noon of the 8th, but coincidentally with the rapid fall of the barometer it increased in violence until it attained hurricane velocity about 5:00 P.M. At 6:15 P.M. the wind attained a velocity of 100 miles per hour, and at this time the wind-registering apparatus was blown away. Between 6:00 and 8:00 P.M. it is estimated that the wind attained a velocity of 120 miles per hour.

At 3:00 P.M. on the 8th, half the streets of the city were flooded, and the water was still rising. It rose steadily till 7:30 P.M., when there was a sudden rise of about 4 feet in as many seconds. This was followed by a more gradual rise up to 8:30 P.M., when the water stood about 15 feet above the level of mean high water. This appears to have been the crest of the rise and from this time the water began to fall.

On Sunday, September 9, the sun rose over a scene of indescribable devastation and ruin. Communication with the mainland was cut off, and the receding waters of the Gulf left in their wake some 6,000 human victims and the shattered remnants of thousands of habitations. The property damage was estimated in tens of millions of dollars. The whole world was shocked and hastened to offer its sympathy and help. Again the people of Galveston accepted the challenge of the sea and now a greater Galveston faces the Gulf, protected by a splendid concrete sea wall 17 feet high and nearly 3 miles long, and the ships leaving its wharves sail from the leading cotton-exporting port in the world.

It will be recalled that the response of the water level to the sudden change in barometric pressure shown in Figure 39 consisted of an oscillation with a range of a little more than half a foot. But what is more striking is that this oscillation continued for several hours after the barometric pressure had become normal. Such oscillations are frequently found on tide curves, especially those representing the rise and fall of the tide in bays or other nearly landlocked bodies of water. It is evident that such oscillations are of a different character from the ordinary waves or swells, the traces of which are indicated by "saw teeth" on tide curves.

The surface of the sea, even in protected bays and harbors, is rarely free from waves. To free the tide curve

from the irregular fluctuations of such waves, which, it is to be noted, are of short period, the float of the automatic tide gauge is confined within a float well to which the tide has access only through a small hole well below the surface of the sea. The float well, which is generally a wooden box or an iron pipe, is about 1 foot in diameter, while the inlet for the tide is about an inch in diameter. This arrangement makes it impossible for waves of short period to cause any appreciable fluctuation within the float well, and it is only waves or oscillations of relatively long period that will manifest themselves on the tide curve.

The short "saw teeth" which the tide curve of Figure 39 shows for the first few hours of the day are undoubtedly due to the ocean waves and swells that enter San Francisco Bay through the Golden Gate. But the larger fluctuations of the water level exhibited on the tide curve between 4:00 and 10:00 A.M. may be assumed to be, from the nature of the case, oscillations peculiar to the Bay itself. Such oscillations are known as seiches. We may study these seiche movements more easily if we separate them from the rise and fall of the tide. This we may do by drawing a smooth curve through the tide curve of Figure 39 and then measuring the seiches from this smooth curve as the axis of the oscillation. The resulting curve exhibiting the seiches free from the rise and fall of the tide from 4:40 to 9:20 A.M., is shown in Figure 40.

If the curve of Figure 40 were that of a simple sine curve, it would be evidence that the oscillation of the water in San Francisco Bay brought about by a sudden change in barometric pressure is of a simple kind. But Figure 40 shows a curve departing considerably from the regularity of a simple sine curve. Hence we are compelled to assume that several simple oscillations are involved. Notwithstanding this, we can see that the predominant oscillation

is one having a period of about three-quarters of an hour and a range of a little over a third of a foot.

Now the interesting feature of the seiche movement in San Francisco Bay is that, no matter what the exciting cause of the seiche may be, the predominant oscillation will have a period of approximately three-quarters of an hour. Thus the great sea waves that frequently result from earthquakes bring about seiches in San Francisco Bay which have a period of about three-quarters of an hour. On August 27, 1883, occurred the terrible earth-

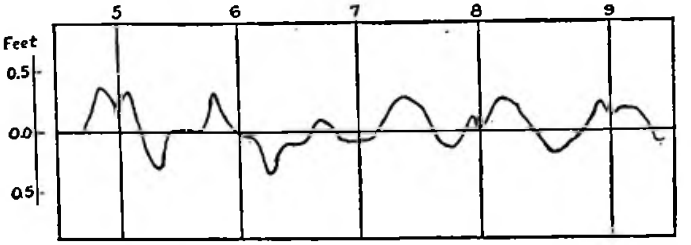


FIG. 40.—SEICHES FREE FROM TIDE, SAN FRANCISCO BAY, NOVEMBER 21, 1910

quake on the island of Krakatoa, situated on the eastern rim of the Indian Ocean. San Francisco is more than 10,000 miles distant from Krakatoa, yet 18 hours after the earthquake, the tide curve for San Francisco shows a series of seiches much like those of Figure 39, the predominant oscillation having a period of about three-quarters of an hour and a range of somewhat more than half a foot. On November 11, 1922, an earthquake occurred off the coast of Chile, 5,000 miles from San Francisco; 14 hours later the tide curve for San Francisco shows a seiche movement with a range more than half a foot and a period of about three-quarters of an hour.

In enclosed bodies of water seiches had been observed and studied for a number of years. The classical work in

this field was done between 1870 and 1880 by F. A. Forel, a Swiss physician who studied the seiches on the Lake of Geneva. Forel proved conclusively that in a landlocked body of water the seiche consists of a rocking of the whole body of water about one or more nodal lines, as illustrated in Figures 30 and 36. In the beginning of the present century, the mathematical theory of the seiche in enclosed bodies of water was developed to a high degree by George Chrystal, an English mathematician, so that now it is possible to calculate with considerable accuracy the period of the seiche in any lake, no matter how irregular in outline, provided the details of its depths and shape are known.

However, it was only toward the end of the first decade of the present century that the theory of the seiche was extended to include bodies of water opening at one end into a much greater basin, as exemplified by bays connected with the ocean. In 1908 there appeared, independently, two publications in which this subject was treated. R. A. Harris, the American tidal mathematician, investigated the matter in connection with his tidal studies. In Japan, Messrs. Honda, Terada, Yoshida, and Isitani attacked the problem in connection with earthquake studies. From these investigations it developed that bodies of water opening at one end into larger basins are capable of sustaining stationary-wave movements of the character exemplified by Figure 30, with this difference, that, instead of the node being located in the middle of the body of water, it was located at the opening into the larger body. That is, the seiche in a bay or other body of water open to the sea would be exemplified by the movement of the water in half of the tank shown in Figure 30.

The seiche therefore belongs to the same type of movement as that exemplified on a larger scale by the tidal movement in Long Island Sound, which was discussed in

Chapter VIII. In fact, the formula for the period of a seiche in any given bay derived by Harris and the Japanese is the same as that made use of in the discussion of the tide in Long Island Sound, namely $T=\frac{4L}{\sqrt{gh}}$, where T is the period of the seiche, L the length of the bay in the direction of oscillation and g and h as before the acceleration of gravity and the depth of the water respectively. In an elongated bay there may exist seiche oscillations with two, three, or more nodal lines, but a factor depending on the number of nodal lines makes the above formula applicable even in such cases.

It is to be observed that the results derived by Harris and the Japanese investigators were based on theoretical considerations. But the Japanese investigators went a step further and constructed models of various bays and studied the actual movement of the water in these miniature bays. They made an accurate model of San Francisco Bay and placed it in a small pond, and by means of a heavy pendulum they excited waves in the pond of any desired period. They found that a seiche in the miniature bay was most easily brought about by waves having a period that corresponded to three-quarters of an hour for the actual dimensions of San Francisco Bay, and that the movement of the water was of the stationary-wave type with a nodal line at the mouth of the bay.

The conclusions resulting from these investigations may be summed up as follows. The seiche is a stationary-wave movement, the period of which depends on the length and depth of the body of water. The exciting cause of the seiche may be sudden changes in barometric pressure, strong winds, or waves from the sea. In the open sea, waves of various periods appear to exist and as these proceed toward the coast the one whose period approximates to the period of the body of water opening into the sea will bring about a seiche in that body of water.

Seiches occur most frequently in deep bays or estuaries that are nearly landlocked. But they may also be found in any portion of the sea partly bounded by land, for any such area is capable of sustaining stationary-wave oscillations of a period conditioned by its length and depth. Thus well-developed seiches with a period of about a quarter of an hour are found on the tide curves for Atlantic City, on the open coast of New Jersey. These may be taken to represent an oscillation of some part of the wide embayment of the coast, the headlands of which are Martha's Vineyard and Nantucket Island on the north and Cape Hatteras on the south.

Before leaving the subject of seiches, mention should be made of the fact that large seiche movements frequently occur on the shores of oceanic islands. An example of this is found in Honolulu, the capital of the Hawaiian Islands, which faces a small harbor on the island of Oahu. The tides here are very small, the average rise and fall being but little over a foot. Seiches, however, have been observed with a range twice as large. These seiches are generally found to be due to an earthquake or submarine explosion at some distant point. The Krakatoa earthquake of 1883 showed well-marked seiches, with a range up to three-quarters of a foot, on the Honolulu tide record. An unusually good example of a seiche at Honolulu is shown in Figure 41, which is a reproduction on a reduced scale of the tide gauge record for February 2-6, 1923.

For February 2, 1923, the first day shown in Figure 41, the tide curve is typical of Honolulu and exhibits traces of moderate swells. This continues to a little before noon on the 3rd when there is a sudden rise of the water of three-quarters of a foot and is followed by a violent oscillation with a period of about a third of an hour, which, between 1:00 and 2:00 P.M., has a range of very nearly 3 feet—more than twice as great as the range of the tide. The

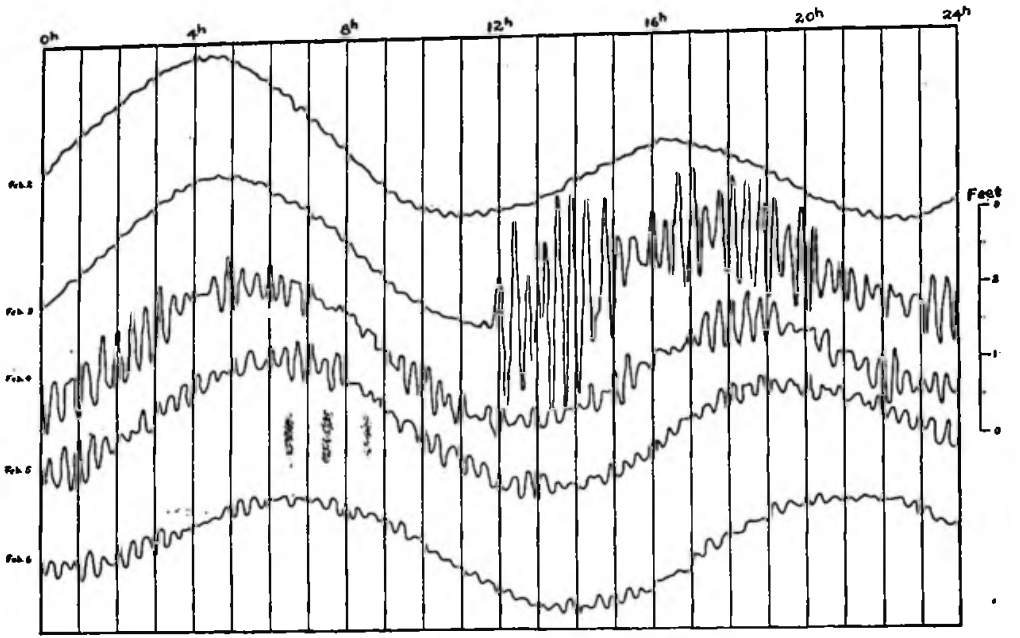


FIG. 41.—TIDE CURVES, HONOLULU, FEBRUARY 2-6, 1923

cause of this seiche movement is found in the earthquake which occurred on February 3, 1923, off the coast of Kamchatka, about 2,500 miles from Honolulu, the first sea wave arriving at Honolulu 6 hours after the earthquake. The tide curves for the following three days show a gradual diminution of the seiches.

Thus far in our discussion of the effects of wind and weather we have confined ourselves to a consideration of these effects on the rise and fall of the tide alone. Now we must consider what these effects are on the currents. And it must be stated at once that as manifested along the coast or in inland tidal waters these effects are not nearly so striking as in the case of the tides. This is due to the fact that along the coast and in inland bodies of water the scope of the wind is restricted. That is, the currents here are either weak or are compelled to follow definite channels. In general, it is easy to see that a wind from the sea will increase somewhat the velocity of the flood current and decrease that of the ebb, while with a wind from the land conditions will be reversed.

Away from the coast, however, the wind has greater scope and here some interesting features have been discovered in recent years. The wind has always been recognized as one of the principal causes of nontidal currents and it has been taken for granted, especially by the mariner, that a wind blowing steadily over a wide stretch of water gives rise to a current which sets with the wind. In this opinion the seafarer received support from an elaborate mathematical investigation of the question published in 1878 by Karl Zöppritz, a German scientist. From this investigation it was concluded that a steadily blowing wind will, through friction, bring about a surface current in the direction of the wind; this surface layer will in turn put into motion, in the same direction, a lower layer of water until finally (after a very considerable period of

time) the whole mass of water, from top to bottom, will be flowing in the direction of the wind. Furthermore, in this moving mass of water, the equations showed that the velocity would decrease in direct proportion to the distance below the surface.

These conclusions resulting from Zöppritz's mathematical analysis were accepted for many years; for these conclusions were in accord with the practical view of the matter and appeared also in accord with such observational material as was at hand. It is to be noted, however, that because of the difficulty and expense involved in securing systematic current observations at some distance from the shore very little observational material of a systematic nature was at hand. It should be stated, too, in passing, that in this mathematical investigation Zöppritz did not include in his equations the effect of the earth's rotation, the assumption undoubtedly being that since the velocities of wind-driven currents were never very great, the effect of the earth's rotation would be so small that it might be completely disregarded.

It was only at the beginning of the present century that attention was directed anew to the matter. At that time the Norwegian oceanographer Fritdjof Nansen was studying the results of his observations made while the *Fram* was drifting across the polar sea. He was struck by the fact that the ice did not drift in the direction of the wind but to the right of that direction. He suspected that this deviation of the ice drift to the right of the wind was due to the earth's rotation, and he therefore requested the Swedish mathematician V. W. Ekman to investigate the matter mathematically.

It has been known for half a century or more that the rotation of the earth gives rise to a force which tends to deflect a moving body to the right in the northern hemisphere and to the left in the southern hemisphere. This

force is known as the deflecting force of the earth's rotation. In his investigation of the current brought about by a wind Ekman introduced this deflecting force of the earth's rotation into his equations, and on the assumption of a large body of water of infinite depth, he arrived at the following results: (1) The effect of the wind is to cause a current which deviates to the right of the wind in the northern hemisphere and to the left in the southern hemisphere; (2) the deviation of the current from the direction of the wind increases uniformly from the surface downward, so that there is a certain depth at which the direction of the wind-driven current is directly opposite to the current at the surface; (3) the velocity of the current decreases from the surface downward, but in geometrical progression, so that at the depth where the current is opposite the direction of the surface current the velocity is only 4 per cent that at the surface; (4) the surface currents due to the wind set in a direction 45° to the right of the wind in the northern hemisphere and 45° to the left in the southern hemisphere; (5) the velocity of the current for different places varies inversely as the square root of the sine of the latitude.

The surprising result from Ekman's investigation is that the deviation of the wind-driven current from the direction of the wind does not vary with the latitude, but is invariable —always 45° at the surface. And since this deviation is to the right in the northern hemisphere and to the left in the southern hemisphere, this result demands an abrupt change in the direction of the wind-driven currents of 90° at the equator. It is to be noted, however, that this result holds only for an ocean of infinite depth, and for the existing oceans the angle of deflection would begin to decrease near the equator and be zero at the equator. In fact, for an ocean of finite depth, Ekman's equations show that the angle between the surface current and the wind

depends on the depth. In a very shallow ocean the angle is very small, that is, the current sets nearly in the direction of the wind; as the depth of the ocean increases, the angle increases and approximates to the value of 45°. From general considerations it would appear that near the shore the direction of the wind-driven current must be very considerably modified by the direction of the coast line. From his investigation Ekman found this to be the case, calculation showing that while the surface current should still deviate to the right of the wind (in the northern hemisphere), this deviation would vary from 0° to 53°, depending on the angle between the coast line and the direction of the wind.

Here again we have results derived from theoretical considerations by means of mathematical processes. The question may therefore be raised, How far are these results borne out by actual observations? There is at hand now a considerable mass of observational material by which some of the results enumerated above may be tested, especially the deviation of the wind-driven current from the direction of the wind.

In the open sea where the depths are reckoned in miles the systematic observation of currents is a matter of such great difficulty that none has as yet been made. But a great deal of information regarding the currents there may be obtained from the observations made by the navigator in charting the course of his vessel across the trackless sea. Each day at noon the position of the ship is determined by means of astronomical observations. The distance steamed by the ship between these noon observations is also determined by means of a patent log or by means of the number of revolutions of the propeller. Knowing the course steered, the navigator calculates the position his ship should be in if it were unaffected by currents, this position being determined, as he styles it, by "dead reckoning." For each

day therefore the difference between the dead reckoning position and that determined by the astronomical observations gives the direction and velocity of the current for the preceding 24 hours. The currents determined in this manner for various tracts of the ocean both in the northern and southern hemispheres were compared with the winds and it was found that in the northern hemisphere the current set about 45° to the right of the wind and in the southern hemisphere about 45° to the left.

Along the coasts of the United States systematic current observations have been made on the light vessels by the Coast and Geodetic Survey. The coast line introduces a complicating factor into the problem for it obviously must modify in different ways the current produced by winds from different directions. In general, however, the observations on both the Atlantic and Pacific coast light vessels show unmistakably that the wind-driven current deviates to the right of the wind. In fact, a general rule has been formulated for the benefit of the mariner which states that along the Atlantic and Pacific coasts of the United States, say from 5 to 20 miles offshore, the velocity of a current produced by a wind is about 2 per cent that of the wind and sets about 20° to the right of the wind.

CHAPTER XII

TIDE TABLES

In ancient times there was little or no need of tide tables, and even in more recent times the navigator troubled himself but little about accurate tidal data, for ships were still of relatively shallow draft and ran on schedules subject to the saving clause "weather and tide permitting." Now, however, with the increased size and draft of modern vessels, operating on exacting schedules, an advance knowledge of the state of the tide is of considerable importance at many places, since a change in the depth over the bar of but a few feet may mean the difference between getting a modern leviathan into port immediately or being compelled to wait several hours for a favorable tide.

Tide tables giving advance information relative to the state of the tide are of importance not only to the mariner but to many others connected with the varied activities of a modern port. The engineer engaged in harbor improvements or in the building of wharf structures, the shipbuilder, the diver, the fisherman—these likewise find tide tables of use in their respective callings. The geographer who is studying the topography of the shore line, the geologist studying its geological structure, and the zoölogist collecting marine specimens frequently have occasion to consult the tide tables to ascertain the time of low water on a given day, or the date of lowest low water when a wide expanse of shore line is exposed for examination. And even in spheres far removed from industry and science tide tables are used. The yachtsman, the amateur fisher-

man, the motor-boat enthusiast, and the seaside bather are also interested in having at hand advance information of the state of the tide.

In time of war accurate tide tables became a matter of prime importance. Take the case of the navigator who, because of lack of advance knowledge relative to the state of the tide at the entrance to a port, finds it necessary to wait several hours for a favorable tide to enter port. While waiting, he exposes his vessel not only to possible bad weather but also to the more serious hazards of submarine attack. With an advance knowledge of the state of the tide at any given port the navigator can time the arrival of his vessel to coincide with a favorable tide at the entrance.

In time of peace a vessel generally follows the customary safe lanes of travel. In war time, when chased by the enemy, the commander of a vessel may frequently save his ship by taking advantage of a short cut over a shoal. He can do this all the more readily when equipped with accurate tide tables so that he may estimate within a foot or two the depth of the water at any time.

During the World War the allied war vessels engaged in the bombardment of the enemy on the Belgian coast were exposed to torpedo attack from the sea. But it was found that, by anchoring in the shoalest water possible, considerable security was afforded since if a torpedo was fired it was more likely to strike the shoal than the ship. Now on the Belgian coast the tides have a rise and fall ranging from 10 to 16 feet. Rough estimates of the height of the tide would not answer the purpose; accurate predictions based on definite knowledge were necessary.

It is this accurate advance knowledge of the state of the tide that a tide table aims to furnish. For any given port a tide table usually presents this information in the form of predicted times and heights of high and low water for every day of a calendar year and is generally published

six months or more in advance of the year for which the tide table is to be used. Each of the leading maritime nations now finds it essential to publish annual tide tables for the use of its navy and merchant marine. To mention only the larger countries beside our own, we find tide tables published by the governments of Great Britain, France, Japan, and Germany.

The oldest tide table of which there is record is one now in the library of the British Museum. It is a manuscript table which appears to have been written in the Thirteenth Century and gives the time of "flod at london brigge," that is, the time of high water at London Bridge. It differs radically from modern tide tables in that the time of tide is given not for calendar days of the month of any particular year, but only with reference to the moon's age.

Of the modern tide tables the first to be published by any government were those issued by the British Admiralty in 1833. Prior to that time local tide tables had been published for several ports privately by enterprising persons. These necessarily kept their methods of making tidal predictions secret, handing the secret down from father to son. In 1839 the French Hydrographic Service began the annual issue of its tide tables and in 1853 the first tide tables of our own government were issued by the Coast and Geodetic Survey.

The appearance of a modern tide table is shown by a the U. S. Coast and Geodetic Survey. This page gives for *States and Foreign Ports, for the Year 1925,* published by specimen page reproduced from the *Tide Tables, United* San Francisco, California, the predicted times and heights of each high and low water for the last three months of the year 1925, four such pages giving data for a whole year. The asterisk in the high-water column of October 2, for the explanation of which reference to paragraph 4, page 7, is made at the bottom of the page, merely calls

OCTOBER.					NOVEMBER.					DECEMBER.				
Day.	High Water.		Low Water.		Day.	High Water.		Low Water.		Day.	High Water.		Low Water.	
	Time.	Ht.	Time.	Ht.		Time.	Ht.	Time.	Ht.		Time.	Ht.	Time.	Ht.
	h. m.	ft.	h. m.	ft.		h. m.	ft.	h. m.	ft.		h. m.	ft.	h. m.	ft.
1	10 59	5.7	4 26	0.1	1	0 06	4.7	5 15	1.8	1	0 55	4.5	5 26	2.7
Th	11 10	5.5	4 51	0.9	Su	11 30	5.8	6 13	−0.4	Tu	11 29	5.6	6 56	−0.3
2	11 36	5.7	5 07	0.5	2	1 01	4.6	5 54	2.2	2	1 46	4.4	6 09	2.9
F	(*)	(*)	5 40	0.6	M	12 03	5.6	6 55	−0.4	W	12 00	5.8	7 18	−0.6
3	0 05	5.3	5 48	1.0	3	1 57	4.5	6 36	2.6	3	2 37	4.4	6 54	2.1
Sa	12 12	5.7	6 28	0.2	Tu	12 35	5.8	7 37	−0.8	Th	12 29	5.1	7 52	−0.4
4	1 02	5.0	6 28	1.5	4	2 54	4.4	7 21	3.0	4	3 27	4.4	7 41	3.3
Su	12 49	5.6	7 17	0.2	W	1 07	5.0	8 20	−0.1	F	1 01	4.8	8 34	−0.2
5	2 00	4.6	7 09	2.1	5	3 54	4.3	8 11	3.2	5	4 15	4.4	8 36	3.2
M	1 26	5.3	8 04	0.3	Th	1 42	4.7	9 07	0.1	Sa	1 37	4.6	9 16	0.1
6	3 03	4.4	7 51	2.5	6	4 53	4.3	9 09	3.4	6	5 02	4.4	9 38	3.2
Tu	2 03	5.1	8 54	0.4	F	2 26	4.4	10 00	0.3	Su	2 29	4.1	9 59	0.3
7	4 10	4.3	8 41	2.9	7	6 47	4.4	10 21	3.4	7	5 42	4.5	10 48	3.0
W	2 45	4.8	9 50	0.5	Sa	3 32	4.1	10 53	0.4	M	3 47	3.8	10 44	0.6
8	5 17	4.2	9 40	3.2	8	6 33	4.5	11 38	3.2	8	6 22	4.7	11 57	2.6
Th	3 38	4.6	10 48	0.6	Su	4 55	4.0	11 44	0.6	Tu	5 15	3.6	11 30	0.9
9	6 20	4.3	10 52	3.3	9	7 14	4.6	. .	. .	9	6 55	4.9	. .	. .
F	4 43	4.4	11 51	0.6	M	6 14	4.0	12 44	2.8	W	6 40	3.6	12 57	2.9
10	7 14	4.4	. .	. .	10	7 49	4.8	0 34	0.7	10	7 29	5.1	0 18	1.1
Sa	5 56	4.4	12 06	3.2	Tu	7 21	4.1	1 36	2.3	Th	7 51	3.7	1 47	1.4
11	7 53	4.5	0 49	0.6	11	8 21	5.0	1 18	0.8	11	7 59	5.4	1 04	1.4
Su	6 59	4.6	1 12	2.9	W	8 18	4.2	2 20	1.7	F	8 52	3.8	2 33	0.7
12	8 36	4.7	1 36	0.6	12	8 51	5.3	2 01	0.9	12	8 30	5.7	1 51	1.6
M	7 55	4.8	2 02	2.6	Th	9 12	4.3	3 00	1.1	Sa	9 50	4.0	3 16	0.1
13	9 09	4.9	2 19	0.6	13	9 19	5.5	2 40	1.1	13	9 03	5.9	2 38	1.8
Tu	8 45	4.8	2 42	2.1	F	10 03	4.4	3 40	0.5	Su	10 45	4.2	4 00	−0.6
14	9 40	5.0	2 58	0.6	14	9 48	5.7	3 20	1.4	14	9 38	6.2	3 23	2.1
W	9 33	4.9	3 22	1.6	Sa	10 53	4.4	4 20	0.0	M	11 39	4.3	4 44	−1.0
15	10 09	5.2	3 31	0.7	15	10 16	5.8	4 00	1.6	15	10 16	6.3	4 10	2.3
Th	10 17	4.9	4 01	1.2	Su	11 45	4.5	5 01	−0.5	Tu	. .	. .	5 29	−1.4
16	10 37	5.3	4 05	0.9	16	10 46	6.0	4 41	1.9	16	0 33	4.4	4 52	2.5
F	11 02	4.8	4 40	0.7	M	. .	. .	5 44	−0.8	W	10 33	5.3	6 16	−1.6
17	11 04	5.4	4 40	1.1	17	0 40	4.4	5 22	2.3	17	1 27	4.5	5 49	2.7
Sa	11 49	4.7	5 19	0.4	Tu	11 18	6.0	6 29	−1.1	Th	11 40	5.2	7 04	−1.6
18	11 29	5.5	5 17	1.4	18	1 36	4.4	6 07	2.6	18	2 21	4.6	6 44	2.8
Su	. .	. .	6 01	0.1	W	11 54	5.9	7 17	−1.1	F	12 29	5.9	7 53	−1.5
19	0 39	4.6	5 52	1.8	19	2 37	4.4	6 56	2.9	19	3 15	4.6	7 45	2.8
M	11 57	5.5	6 44	−0.2	Th	12 34	5.8	8 08	−1.0	Sa	1 25	5.5	8 45	−0.9
20	1 34	4.4	6 32	2.2	20	3 40	4.4	7 53	3.1	20	4 07	4.8	8 55	2.8
Tu	12 26	5.5	7 30	−0.3	F	1 24	5.5	9 02	−0.8	Su	2 31	4.9	9 54	−0.4
21	2 36	4.2	7 14	2.6	21	4 41	4.5	9 00	3.2	21	4 38	4.9	10 16	2.6
W	12 59	5.5	8 23	−0.4	Sa	2 29	5.0	9 59	−0.5	M	3 55	4.3	10 26	0.1
22	3 46	4.1	8 04	2.9	22	5 36	4.7	10 24	3.0	22	5 46	5.2	11 43	2.0
Th	1 41	5.3	9 19	−0.3	Su	3 54	4.6	10 59	−0.2	Tu	5 26	3.9	11 20	0.3
23	4 58	4.2	9 05	3.2	23	6 27	4.9	11 52	2.6	23	6 33	5.4	. .	. .
F	2 38	5.1	10 22	−0.2	M	5 31	4.3	11 57	0.2	W	6 54	3.7	1 08	1.4
24	6 03	4.3	10 22	3.2	24	7 12	5.2	. .	. .	24	7 17	5.6	0 14	1.2
Sa	4 01	4.9	11 27	−0.2	Tu	6 57	4.2	1 18	2.0	Th	8 11	3.7	2 09	0.8
25	7 01	4.6	11 43	3.0	25	7 54	5.5	0 53	0.6	25	8 00	5.8	1 09	1.7
Su	5 35	4.8	. .	. .	W	8 12	4.2	2 18	1.2	F	9 18	3.8	3 04	0.2
26	7 47	4.9	0 31	−0.1	26	8 34	5.8	1 44	1.0	26	8 42	5.8	2 01	2.1
M	7 00	4.7	1 09	2.5	Th	9 20	4.2	3 11	0.6	Sa	10 16	4.0	3 47	−0.2
27	8 29	5.2	1 30	0.1	27	9 11	6.0	2 32	1.4	27	9 20	5.8	2 51	2.3
Tu	8 12	4.8	2 14	1.8	F	10 17	4.3	3 56	0.0	Su	11 08	4.2	4 27	−0.6
28	9 08	5.5	2 22	0.3	28	9 48	6.0	3 17	1.8	28	9 57	5.8	3 39	2.6
W	9 18	4.9	3 10	1.2	Sa	11 12	4.4	4 38	−0.4	M	11 55	4.3	5 04	−0.7
29	9 45	5.8	3 08	0.6	29	10 23	5.9	4 01	2.1	29	10 32	5.7	4 22	2.7
Th	10 16	4.9	3 59	0.6	Su	. .	. .	5 18	−0.6	Tu	. .	. .	5 40	−0.8
30	10 21	5.9	3 51	0.9	30	0 04	4.5	4 44	2.4	30	0 40	4.4	5 05	2.8
F	11 11	4.8	4 46	0.1	M	10 56	5.8	5 57	−0.7	W	11 07	5.5	6 17	−0.8
31	10 57	5.9	4 33	1.4						31	1 25	4.4	5 50	2.9
Sa	. . .	. . .	5 30	−0.2						Th	11 40	5.3	6 51	−0.7

Time meridian 120° W. Heavy-faced type indicates p. m. tides. Heights are reckoned from mean lower low water, the datum of soundings on Coast and Geodetic Survey charts.
* For explanation see par. 4, p. 7.

FIG. 42.—SPECIMEN PAGE OF TIDE TABLE FOR SAN FRANCISCO FOR OCTOBER, NOVEMBER AND DECEMBER, 1925 (REDUCED SOMEWHAT)

attention to the fact that there are usually two high and two low waters each day; but since the tide becomes later each day by about 50 minutes, a tide occurring after midnight of any given day will be found as the first tide of the next day, and but two high waters and one low water, or two low waters and one high water will appear for the day in question.

The morning tides are indicated by the light-faced type, while the afternoon tides are indicated by the heavy-faced type. The heights are reckoned from the plane of mean lower low water, which is the average of all the lower low waters of each day over a considerable period of time. This reference plane is likewise the one which is used on the charts of the Pacific coast, so that the mariner, by the aid of his chart and the tide table, can tell the depth of the water at any point at the time of low water or high water, and by a little computation can determine the depth of the water at any intermediate time. The figures with a minus sign before them indicate that the height given is below the plane of reference.

The predictions for the year 1925 were made in 1923 and the tables issued in 1924, so that before the beginning of the year 1925 these tide tables would be in the hands of the navigator. As at present appearing, the tide tables issued by our government contain the predicted times and heights of the tide for each day of the year for 83 of the more important ports of the world.

It is obvious that space and expense limit the number of ports for which daily predictions of high and low water can be given in the tide tables. To enable the mariner to secure advance tidal information for other ports, the tide tables give the differences in the times and heights of the tide at a large number of places as compared with the tides at the ports for which daily predictions are given. Thus, in addition to the daily predictions for the whole

year at 83 of the principal ports of the world, the Coast and Geodetic Survey tide tables contain also tidal data for more than 3,500 secondary ports, which, in connection with the daily predictions given, enable the navigator to make use of practically any port in the world. As an

New Jersey.	Latitude.	Longitude.	Tidal difference. Time of tide.	Tidal difference. Height of H. W.	Ratio of ranges.	High water interval.	Mean range of tide.	Spring range of tide.
	° ′	° ′	h. m.	feet.		h. m.	feet.	feet.
NEW JERSEY, *Outer Coast.*	*North.*	*West.*	Standard port, Sandy Hook, page 53.			*Time meridian, 75° W.*		
Little Egg Inlet, Sea Haven Light	39 30	74 18	+0 05	−1.4	0.7	7 42	3.3	3.9
Great Bay (entrance)	39 30	74 22	+0 35	−1.8	0.6	8 05	2.9	3.5
Mullica River Entr., Great Bay	39 32	74 24	+1 15	−2.0	0.6	8 34	2.6	3.2
Main Marsh Thorofare	39 29	74 23	+0 50	−1.9	0.6	8 15	2.8	3.4
Simkins Thorofare	39 28	74 22	+0 55	−1.8	0.6	8 24	2.9	3.5
Grassy Bay	39 26	74 24	+1 20	−1.5	0.7	8 45	3.2	3.8
Golden Thorofare	39 24	74 24	+0 55	−1.1	0.8	8 15	3.6	4.3
Absecon Bay	39 24	74 29	+1 10	−0.7	0.8	8 37	3.9	4.7
Absecon Inlet, Gardiners Basin	39 22	74 25	−0 05	−0.9	0.8	7 20	3.8	4.6
Atlantic City (near the bridges)	39 22	74 27	+1 10	−1.1	0.8	8 35	3.6	4.3
Atlantic City, Million Dollar Pier	39 21	74 26	−0 25	−0.6	0.9	7 11	4.0	4.8
Ventnor (inside)	39 21	74 29	+1 20	−1.3	0.7	8 45	3.4	4.1
Persimmon Point	39 21	74 30	+1 05	−1.2	0.7	8 30	3.5	4.2
Risley channel	39 19	74 32	+0 20	−0.9	0.8	7 55	3.8	4.6
Longport (inside)	39 18	74 32	+0 05	−0.8	0.8	7 40	3.9	4.7
Great Egg Inlet	39 18	74 33	0 00	−0.6	0.9	7 36	4.1	4.9
Ocean City, Fourth Street (inside)	39 17	74 35	+0 15	−1.0	0.8	7 50	3.7	4.4
Ocean City, 34th Street (inside)	39 17	74 35	+1 00	−1.7	0.6	8 30	3.0	3.6
Crook Horn Thorofare	39 14	74 39	+0 55	−0.8	0.8	8 25	3.9	4.7
Corson Inlet	39 12	74 39	+0 05	−0.5	0.9	7 35	4.2	5.0
Ben Hands Thoroughfare	39 12	74 41	+0 35	−0.8	0.8	8 05	3.9	4.7
Sea Isle City (inside)	39 09	74 42	+0 50	−0.7	0.9	8 10	4.0	4.8
Townsend Inlet (entrance)	39 07	74 43	−0 05	−0.5	0.9	7 32	4.2	5.1
Leaming Bay	39 05	74 46	+1 05	−1.3	0.7	8 30	3.4	4.1
Stone Harbor (inside)	39 04	74 45	+1 10	−1.4	0.7	8 35	3.3	4.0
Great Channel (inside)	39 03	74 46	+0 40	−1.1	0.8	8 10	3.6	4.3
Grassy Sound Channel	39 02	74 49	+0 30	−1.1	0.8	7 55	3.6	4.3
Hereford Inlet	39 01	74 47	−0 05	−0.4	0.9	7 20	4.2	5.1
Grassy Sound	39 01	74 49	+0 40	−1.1	0.8	8 05	3.6	4.3
Richardson Channel	38 59	74 51	+0 15	−0.7	0.9	7 40	4.0	4.8
Jarvis Sound	38 58	74 52	+0 25	−0.3	0.9	7 55	4.1	5.8
Cold Spring Inlet, Sewell's Point	38 57	74 52	0 00	−0.3	0.9	7 37	4.4	5.3
Cape May	38 56	74 55	+0 15	−0.1	1.0	7 50	4.6	5.5

FIG. 43.—SPECIMEN PAGE OF TIDAL DIFFERENCES FOR COAST OF NEW JERSEY

example of this form of tidal information, there is reproduced here part of a page from the table of tidal differences along the coast of New Jersey, given in the tide tables for 1925.

The standard port to which are referred the tides at all

the stations listed in the above table is Sandy Hook, daily predictions for which are given in the tide tables. By the use of the differences given in such a table, the times and heights of the tide at the places named may be predicted in advance, for any given day of the year for which predictions at the port of reference are at hand. As an example, suppose it is desired to ascertain the times and heights of high and low water at Atlantic City, near the Million Dollar Pier, on a given day. From the table of differences above, we find that as compared with Sandy Hook the time of tide near the Million Dollar Pier is 25 minutes earlier and the height of high water 0.6 of a foot less. Now, upon looking up the predicted times and heights of the tide at Sandy Hook for the given day and applying the above differences, we have the predicted times and heights of the tides desired.

In a table of tidal differences it is the general practice to refer the tides at a number of points to the tides at the nearest principal port for which daily predictions are given. This is based on the fact that, as a rule, the tides at near-by places are of the same type. Where differences in type of tide occur it may be necessary to refer the tides at a given place not to the nearest principal port, but to some distant port which has a similar type of tide. Thus it has been found more satisfactory to refer the tides at a number of islands in Bering Sea, Alaska, to the tide at San Diego, California, rather than to the tide at nearer stations in Alaska, like Sitka or Kodiak.

To predict the time and height of high and low water for the use of tide tables, two different methods have been employed, known technically as the harmonic and nonharmonic methods. The nonharmonic method was the one first used, the harmonic method not being employed until after the introduction of the harmonic analysis. It will therefore be of advantage to take up the nonharmonic

method first, leaving the harmonic method to follow the chapter on the harmonic analysis. For both methods it is necessary to have observations at the port for which the predictions are desired. At least one month of observations are necessary, but a year or more is much better. From these observations are derived the data used in the prediction of the tides.

Essentially, the nonharmonic method is based on the principle that "the tide follows the moon" and makes use of the close relationship existing between the time of tide at most places and the moon's meridian passage. It begins by determining, from the observations made at a port, the intervals elapsing between the moon's meridian passage and the times of high and low water at that port. These intervals, known respectively as the high water and low water lunitidal intervals, have an approximately constant value for any given place and after having been once determined may be used for making a rough tide table for that place by adding to the times of the moon's meridian passage as given in a nautical almanac.

The lunitidal intervals for any given place, as mentioned above, are only approximately constant. During a lunar month they undergo periodic changes, depending principally on the phase and declination of the moon. From long series of tidal observations these periodic changes may be determined, and by using these as corrections to the lunitidal intervals, satisfactory predictions may be secured for many ports. It was by various modifications of the method outlined above that tide tables were computed for many years.

The heights of high and low water were predicted in a similar manner. The average heights for high and low water, at the port for which predictions were desired, were determined from observations. To these average heights corrections were then applied for variations in the phase,

parallax and declination of the moon, these corrections likewise being derived from observations. And the tide tables produced in this way, although only approximately correct, worked quite satisfactorily for tides of the semi-daily type.

An example may make the use of the nonharmonic method clearer. Suppose that it is desired to make a tide table for Sandy Hook, New Jersey, off the entrance to New York Harbor, for the year 1926. From observations made years ago, we find that on the average the times of low water and high water at Sandy Hook follow the times of the moon's passage over the meridian of Greenwich by $1^h\ 36^m$ and $7^h\ 42^m$ respectively. The moon's passage over the meridian of Greenwich is used because the nautical almanacs give the time of this meridian passage.

To predict the times of tide at Sandy Hook for 1926, we therefore provide ourselves with a nautical almanac for that year. (And parenthetically it may here be observed that the nautical almanacs are issued two or more years in advance.) Starting with January 1, we add to the times of the moon's upper and lower transits our interval of $1^h\ 36^m$ for the times of low water and $7^h\ 42^m$ for the times of high water and get a first approximation to the predicted times desired. On consulting the nautical almanac we find that on January 1, 1926, the upper transit of the moon across the meridian of Greenwich will take place at 1:36 A.M., and the lower transit at 2:00 P.M. Adding our lunitidal intervals of $1^h\ 36^m$ for the times of low water and $7^h\ 42^m$ for low water we get as our first approximation the following values for January 1, 1926: low water, 3:12 A.M. and 3:36 P.M.; high water 9:18 A.M. and 9:42 P.M.

The intervals by which the high and low waters at any place follow the moon's meridian passage, while approximately constant, vary throughout the month. In tides of

the mixed type this variation is due chiefly to the changes in the declination of the moon and frequently becomes considerable. For this reason a correction to our first approximation of the predicted times must be applied for this declinational variation in tides of the mixed type. But even in the semidaily type of tide there is some variation in the lunitidal intervals, brought about by changes in the moon's phase. That this is to be expected is evident from the following considerations.

The tide-producing forces of the sun tend to bring high water (or low water) at the same time each day. But the tide brought about by the tide-producing forces of the moon tends to come later each day by 50 minutes. At the times of full or new moon, when the meridian passage of sun and moon takes place at the same time, the solar and lunar tides come to high water or low water at the same time, so that the solar tide at these times neither retards nor advances the time of the actual tide. For the following period of a week the solar tide is earlier each day than the lunar tide, and hence the combined tide actually occurring follows the moon's meridian passage by an interval less than the average. At the end of the week the moon is 7×50^m, or about 6 hours, behind the sun, and the solar low water now corresponds with the lunar high water and hence it is only the height of the lunar tide that is now affected, the time of the tide remaining unchanged, so that the interval between moon and tide at this time is again at its average value.

From full moon to new moon or from new moon to full moon is approximately a fortnight or two weeks. For the week following new or full moon the lunitidal intervals, as we have just seen, are less than the average. What are the lunitidal intervals for the other week of this period? We can answer that most easily by beginning again at the time of new or full moon and working backwards for a

week. At the time of full or new moon, the times of the transits of sun and moon are the same. The day previous the sun's meridian passage occurred 50 minutes later than that of the moon; hence the effect of the solar tide was to make the actual tide somewhat later. And for a period of a week prior to the full or new moon the solar tide occurred later than the lunar tide, so that during this period the lunitidal interval was greater than the average.

The above explanation does not take into account other factors which modify somewhat the effects as stated. In general, however, we may say that for a week following new or full moon the lunitidal intervals are less than the average and for the following week greater than the average. From observations the change in interval for each hour of transit may be determined and the rough predictions made by adding the average lunitidal intervals to the times of the moon's transit corrected for this change. For a transit occurring at 1:36 A.M. or P.M., that is, when the age of the moon is $1^h\ 36^m$, the tidal observations show that at Sandy Hook the time of high water occurs 7^m earlier than the average and the time of low water 11^m earlier; and for a transit occurring at 2:00 A.M. or P.M. the correction is 10^m for high water and 17^m for low water. We can therefore now correct our approximate times of high water for January 1, 1926, by the above corrections and get for the time of low water 3:01 A.M. and 3:19 P.M., and for the time of high water 9:11 A.M. and 9:32 P.M.

The heights of high and low water may be predicted in a similar manner. From the observations made at Sandy Hook, we find that high water, on the average, rises 4.7 feet above low water. Taking the plane of average low water as the plane of reference, our first approximation for the heights of high water is therefore 4.7 feet and for low water 0.0 feet. But a moment's consideration will

show that at full and new moon high water is higher than the average and low water lower than the average, while a week later when the low water of the solar tide corresponds with the high water of the lunar tide, high water is lower than the average and low water higher than the average. A table showing the changes in height of high and low water due to the change in phase of the moon is therefore made out from the observations at Sandy Hook and applied as corrections to our height of 4.7 feet for high water and 0.0 feet for low water. From this table we find that for a transit of $1^h\ 36^m$ the correction for high water is +0.3 foot and for low water —0.3 foot; and for a transit of $2^h\ 00^m$ the corrections are +0.2 and —0.3 respectively. Our heights for January 1, as corrected for the moon's phase, now therefore stand as 5.0 feet and 4.9 feet for high water and —0.3 foot for the low waters.

The heights of high and low water should be further corrected for the moon's parallax and declination, these corrections, too, being determined from observations. But we need not pursue the matter any further, as the outline of the methods employed are probably sufficient for purposes of illustration. It should be stated, however, that the professional tidal computer had developed short cuts in the methods of computation, so that the predictions were made much more expeditiously than illustrated above. It may be of interest to compare our approximate prediction for Sandy Hook with the more correct one made by use of a tide-predicting machine which takes account of all the important variations in the movement of moon and sun. As given in the tide tables published by the U. S. Coast and Geodetic Survey, the times and heights of the tide at Sandy Hook for January 1, 1926, are as follows: Low water 2:56 A.M. and 3:35 P.M.; high water 9:00 A.M. and 9:31 P.M. The heights in feet are given as follows: Low water 0.0 and —0.4; high water 4.7 and 3.8. Our times

of tide are thus, on the average, within 10 minutes of the correct predictions and our heights within half a foot. Had corrections for parallax and declination been applied the agreement would of course have been closer.

How closely the predictions given in tide tables agree with the actual times and heights of the tide is a matter best left to be taken up in connection with the harmonic prediction of the tide. Here, however, attention may be directed to the fact that absolute agreement is quite out of the question, for the times and heights of the tide are modified to a considerable extent by wind and weather. But it is to be remembered that at the time of high and low water the height of the tide is changing very slowly so that a disagreement of as much as half an hour makes little difference for most practical purposes.

For tides of the semidaily type the nonharmonic method of prediction proved quite satisfactory, and it is to be remembered that the earlier tide tables were issued for the coasts of Europe and North America, where the tides are of the semidaily type. When applied to the prediction of the mixed type of tide the nonharmonic method necessitates an increase in the number of corrections to give satisfactory results, but for ports where the tide frequently is of the daily type this method is quite unsuitable. However, when the need arose for accurate tide tables covering the whole maritime world, it was found that the mathematician had already developed, in the harmonic analysis, a powerful method for computing and predicting tides.

In connection with tide tables current tables are frequently issued. The need for these arises from the fact that in some places the time of the slack of the tidal current is to the navigator of very considerable importance, outweighing in importance the times of high and low water. As examples may be cited Hell Gate in the East River of New York Harbor where the tidal current attains a

velocity of 5 knots at times of strength, Sergius Narrows, Alaska, where the velocity of the current is 6 knots or more, and Seymour Narrows, British Columbia, where the velocity is 8 to 10 knots. In such places it frequently becomes necessary for the navigator to await the time of slack water before attempting to make the passage. Obviously it is of advantage to have tables giving the times of slack water, in advance, for a year or more.

For the United States the current tables are issued separately for the Atlantic and Pacific coasts. These current tables resemble tide tables closely. For a number of the more important ports the times of slack water for each day of the year are given; and for other places the differences in time of current are given with respect to these ports. The methods of prediction likewise are similar to those employed in predicting the tides, so that it is not necessary to go into detail regarding the matter. Mention, however, should be made of the fact that, because of the intimate connection between tides and currents, it is frequently feasible to predict the time of slack current from the predicted times of high and low water. But the harmonic method is just as applicable to the currents as to the tides, and we shall therefore now take up the consideration of the harmonic analysis as introductory to the harmonic prediction of the tides and currents.

CHAPTER XIII

HARMONIC ANALYSIS

In considering the rotation of the earth and its movement around the sun, it is frequently of great convenience to treat the matter as if the sun revolved about the earth. This mode of speaking makes it possible to regard both moon and sun as revolving about the earth, and any argument applied to the movement of the one may then be applied to the other. Whenever, therefore, we speak of the movement of the sun about the earth, the apparent movement is to be understood.

If the revolution of sun and moon around the earth could be made to take place in circular orbits in the plane of the equator, with the earth as center, the tide would present a very simple phenomenon. For the lunar tide would be the same day after day and likewise the solar tide; so that each lunar month the combined tide would repeat itself exactly in the same way. The only change in the tide throughout the month would be that due to phase. The tide at any given place would be completely specified by two pairs of figures, one giving the time of the solar tide and its range, and the other the lunitidal interval and range of the lunar tide.

The actual motions of the moon and sun, however, are quite complicated. Instead of moving in circular orbits, the orbits are elliptical, and instead of moving in the plane of the equator, sun and moon move in orbits that are inclined to the plane of the equator, so that sometimes they are north and at other times they are south of the

equator. Now since we cannot change the movements of moon and sun, the harmonic analysis does the next best thing; it substitutes for sun and moon as tide producers, a number of hypothetical suns and moons, which with respect to the earth as center have circular orbits in the plane of the equator. Each of these hypothetical suns and moons is considered as bringing forth a tide of its own, the high water of which occurs a certain number of hours after its upper meridian passage, and the low water the same number of hours after its lower meridian passage. The actual tide at any place is thus regarded as being made up of a number of simple tides, each of which is due to a hypothetical tide-producing body.

In the harmonic analysis of the tide the mathematician is not concerned with substituting for sun and moon a number of simpler suns and moons whose net effects would be the same as those of the actual sun and moon. He is fortunately not compelled to tackle that problem, and he is altogether free from the necessity of concerning himself with the distances, masses, or mutual attractions of the hypothetical tide-producing bodies that he invokes. All that he need consider is their speeds of revolution about the earth, or the time taken to complete a revolution. In choosing these assumed suns and moons or hypothetical tide-producers, the mathematician is bound by one condition only, and that is, that their combined tide must be the same as the actual tide of sun and moon.

We have found that on the average the moon brings about two complete cycles of high and low water in a lunar day of $24^h\ 50^m$, giving the average tidal cycle a period of $12^h\ 25^m$. We may therefore head our list of hypothetical tide-producing bodies by assuming one which with a uniform velocity revolves around the earth in a circular orbit with a period of $12^h\ 25^m$. At any given place this hypothetical tide producer is assumed to bring

about a simple tide, the high water of which comes a certain number of hours after its upper meridian passage and the low water the same number of hours after its lower meridian passage. The interval between its meridian passage and the high water of its simple tide, and also the range, differ from place to place and can be determined only from tidal observations. But its time of transit, or its place in the heavens, and also its speed are fixed.

Each of the simple tides into which the tide of nature is resolved by the harmonic analysis is called a "harmonic component tide" or simply a "component." And it is convenient to use the same term to designate the hypothetical tide-producing body which gives rise to a simple tide; that is, we may speak of a simple tide or its tide-producing body as a component. The various components have been given names to distinguish them from each other. Our first one, with a period of $12^h\ 25^m$ is called the "principal lunar semidiurnal component," which tells by its name its complete history, namely, that it is a component having a period of half a day, that it is a lunar component, and of its class the principal one. But it is a rather long name and it can be more conveniently represented by a symbol which has been agreed upon in all languages, namely M_2, the M indicating its derivation from the moon and the subscript 2 that it is a semidaily component.

The M_2 tide therefore will give us high water and low water following each other regularly every $6^h\ 12\frac{1}{2}^m$ with uniform ranges, and it is to be noted that this represents the average semidaily lunar tide. But the range of the lunar tide changes from day to day in accordance with the changing distance of the moon from the earth. In its revolution around the earth the moon describes an ellipse in a period of 27.55 days, this being the average period from one perigee to another. To take account of this

change we introduce another component with such a period that at perigee its high water will correspond with the M_2 high water, and at apogee its low water will correspond with the M_2 high water. This will make the combined range at perigee greater than the average and at apogee less than the average. It therefore follows that this new component, which is to take account of the varying distance of the moon from the earth, must have such a speed that in a period of 27.55 days it will lose a complete revolution on the M_2 component.

Since $12^h\ 25^m = 12.42$ hours, our M_2 component will, in 27.55 days or 661 hours, complete $661 \div 12.42 = 53\frac{1}{4}$ revolutions. Our new component must therefore in this length of time complete $52\frac{1}{4}$ revolutions, giving for its period $661 \div 52\frac{1}{4} = 12.65$ hours, or $12^h\ 39^m$. This component is known as the "larger lunar elliptic semidiurnal," but it is generally called by its symbol N_2.

We now have two components, one representing the mean lunar tide, and the other the effect of changes in the moon's distance. How are the effects of the changes in declination of the moon to be represented? Certain considerations which enter into the problem, with which we need not here concern ourselves, make it more convenient to take account of the declinational effect by the use of two components rather than one. These two components are both called lunar diurnal components and are represented by the symbols K_1 and O_1, the subscript 1 indicating that these are diurnal components. The periods of revolution of these two components may be derived by reasoning similar to that employed for M_2 and N_2.

To begin with, we learn from astronomy that the average period from one maximum semimonthly declination of the moon to another is half a tropic month, or 13.66 days. And from our study of types of tide, it is obvious that in order to produce diurnal inequality in the mean lunar tide

having a period of $12^h\ 25^m$, we must introduce a tide having a period twice as long. So that K_1 and O_1 will have periods the average of which is $24^h\ 50^m$ or 24.83^h. When the moon is on the equator the declination effect, as we have found, is a minimum; and when she is at her maximum semimonthly declination, the declinational effect is a maximum. Hence we must so choose the periods of K_1 and O_1, that when the moon is on the equator they will neutralize each other, that is the low water of one must occur at the time of the high water of the other, and when the moon is at its maximum semimonthly declination they will reinforce each other to produce the maximum declinational effect. We must therefore give K_1 and O_1 such periods that in 13.66 days or 327.84 hours, K_1 will gain a complete revolution on O_1. If K_1 and O_1 each had a period of $24^h\ 50^m$ or 24.83^h they would, in 327.84 hours, go through $327.84 \div 24.83 = 13.2$ revolutions; but since during this time K_1 must gain one revolution on O_1, we have K_1 completing 13.7 revolutions and O_1 12.7 revolutions during 327.84 hours. Simple division now gives for K_1 a period of $23^h\ 56^m$ and for O_1 a period of $25^h\ 49^m$.

Additional components are introduced to take account of other variations in the moon's motion, but the periods of all the lunar harmonic components are derived from astronomical considerations similar to those employed in the case of M_2, N_2, K_1, and O_1. These four components, however, are the principal lunar components, and, as we shall see, these four together with the principal solar component are sufficient for specifying the principal characteristics of the tide at any given place.

For the sun it is evident that the principal solar harmonic component must be one which will give two high and two low waters every 24 hours. Hence the period of this component which is designated by the symbol S_2, is exactly 12 hours. The declinational effects of the sun, like those of

the moon, are taken account of by two diurnal components, but since these are not as important as the lunar diurnal components we need not pursue this phase of the subject any further.

Theoretically, to represent the tide as it actually occurs, a very large number of simple tides is required. At first glance, therefore, it would appear as if the harmonic analysis really complicated matters. But it is to be noted that the ranges of the greater number of these simple tides which theory requires are so small—less than one one-hundredth of a foot—as to be practically negligible. In fact the tide at any place, however complicated, can be represented with a very high degree of precision for practical purposes by about twenty component or simple tides.

The basis of the harmonic analysis thus lies in the conception of the tide as the sum of a number of simple tides each of which has a definite period that is determined by some motion of the moon or sun relative to the earth. In the rise and fall of the tide as it actually occurs at any place, the characteristics of the simple tides conceived by the harmonic analysis are merged together in apparently inextricable fashion. How can we, from the actual tide, separate out the individual simple or component tides?

Before entering into the details of the methods employed it will be of advantage to consider somewhat more fully the characteristics of a simple tide. Let us assume a sea in which a simple tide having a period of 12 hours rises and falls with a range of 2 feet. For the zero hour of our time let us take the instant when the hypothetical tide-producing body that gives rise to this simple tide passes over the local meridian. In Figure 44, the rise and fall of this simple tide is shown with the further assumption that high water comes at 2:00 o'clock. The vertical lines are

spaced an hour apart and thus by their lengths represent the height of the tide at each hour.

The curve of Figure 44, which represents the rise and fall of a simple tide, is known to the mathematician as the cosine curve, and it is therefore no more than fair to adopt his language with regard to the characteristics of this curve. The vertical distance from the mean sea-level line to the highest or lowest point of the curve is called the *amplitude* of the tide (in our figure represented by the height at 2:00 o'clock and the depth at 8:00 o'clock). Obviously the amplitude is half the range of the tide,

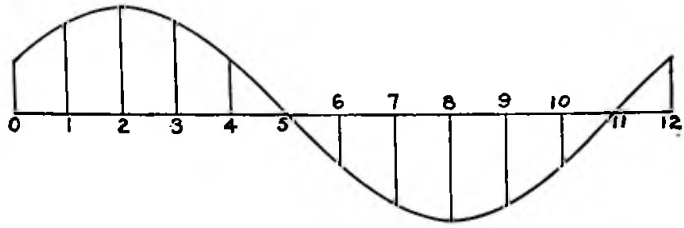


FIG. 44.—TIDE CURVE OF SIMPLE TIDE

since the curve is symmetrical with respect to the mean sea-level line. It is to be noted, too, that each half of the curve is symmetrical with respect to the amplitude, and that any phase of the curve is repeated in reversed position on the opposite side of the mean sea-level line 6 hours later.

For purposes of computation the mathematician has found it convenient to express the period of the curve in degrees as well as in hours, a full period always comprising 360°. In order to change the time of any point on the curve into degrees, obviously all that is necessary is to divide 360° by the period in hours and then multiply by the time of the desired point. Thus in our figure the time

when the water will first reach sea level may be expressed either as 5 hours or as $(360° \div 12)5 = 150°$. The interval elapsing between the time of meridian passage of a hypothetical tide-producing body and the high water of its tide is of special significance and has been named the *epoch* of the simple tide. In our figure, since each hour represents 30°, the epoch is 60°.

Now these two characteristics of a simple tide, the amplitude and the epoch, completely specify the tide. Knowing these two we can construct, with the aid of an ordinary table of cosines, the curve of the simple tide with respect to height and time. The amplitude and epoch of a simple tide, together, constitute the tidal constants for that tide; collectively, the amplitudes and epochs of the various simple tides are known as the harmonic constants. And it is customary to designate the amplitude of any simple or component tide by the symbol of the component and the epoch by the symbol with a degree mark added. Thus, in New York Harbor, at the mouth of the Hudson, the principal lunar semidiurnal component has an amplitude of 2.11 feet and an epoch of 233°; symbolically this would be expressed, $M_2 = 2.11$ feet, $M_2° = 233°$. In the harmonic analysis of the tide we are therefore concerned with determining the amplitudes and epochs of the individual component tides.

Suppose the tide at any place to be an ideally simple tide, corresponding, let us assume, with the principal solar semidiurnal tide. Then the determination of its amplitude and epoch would be a simple matter. For all that would be necessary would be to plot the height of the tide as observed every hour for a period of 12 hours and draw a curve through the plotted heights. The highest point in the curve as measured from mean sea level would then give us the amplitude, and the time of this point as measured in degrees from the instant of the meridian passage

of the hypothetical sun would give us the epoch. And parenthetically it may be remarked that the times of meridian passage of the various hypothetical tide-producing bodies have been calculated for many years in advance and may be found in various tidal manuals.

Let us now assume a tide made up of two simple tides, one having a period of 12 hours and the other a period of 5 hours. We may assume further that a record of the rise and fall of this tide has been made by an automatic tide gauge for a number of days so that the height of the tide at any instant may be determined. It is to be noted, however, that after an interval of 2½ days or 60 hours, conditions will be exactly the same as at the time of beginning, for the first simple tide will have completed 5 full periods and the second tide 12 full periods. Fixing now our attention on the 12-hour tide, it is obvious that if we could measure its height alone at the beginning of each 12-hour period, or at its zero hours, we would get the same heights; but as measured from the record of the automatic tide gauge we get different heights, for at some of these zero hours the 5-hour tide is above mean sea level and at other times below. The important point to note, however, is that if we take the average of all the heights of the tide as observed at the beginning of each 12-hour period over the space of 60 hours, *the effect of the 5-hour tide will be completely eliminated.* And similar reasoning shows that the same holds for the 1-hour, 2-hour, etc., of the 12-hour periods. Plotting now these average hourly heights we get a curve from which the amplitude and epoch of the simple wave is determined as in the preceding paragraph.

To determine the amplitude and epoch of the 5-hour tide we proceed in a similar manner. Periods of 5 hours are taken and the first, second, third, fourth, and fifth hours of the 12 periods constituting the 2½ days are averaged, the curve drawn and the amplitude and epoch

determined. For a tide composed of three simple tides having periods say of 5, 7, and 12 hours, the procedure would be first to secure observations covering a period of $5\times7\times12=420$ hours or 17½ days. To determine the tide of 5-hour period we would divide the tidal record for the 17½ days into periods of 5 hours and from these determine the average height for each of the 5 hours of the period. For the 7-hour tide the periods into which the tide is divided would cover 7 hours and for the 12-hour tide 12 hours. The principle is now clear and can be applied to disentangle from the tide observed at a given place any number of simple tides, provided their periods are known. And it is to be recalled that the periods of the simple tides are determined once for all from the known movements of sun and moon relative to the earth. In practice, it should perhaps be stated, the procedure outlined above is modified to ensure greater accuracy and also to reduce to a minimum the laborious computations involved.

The harmonic analysis of the tide is due to the brilliant English mathematical physicist William Thomson, better known as Lord Kelvin, who first proposed it in 1867. It is the most powerful method of analyzing the tides, for it is applicable to all types of tide, and by resolving the tide into its simple constituents it permits a better understanding of the behavior of the tide.

From the harmonic constants of the tide at any place we can determine the characteristics of the tide at that place. Formulas have been developed for deriving from the harmonic constants such characteristics as the spring, neap, apogean, and perigean ranges; the lunitidal intervals, the age of the tide and many others. These formulas, to give the accuracy required by the tidal mathematician, are rather elaborate; but, approximately, we may determine the principal tidal characteristics very easily from the five principal components, the periods of which were derived

in this chapter. These components are M_2, the principal lunar semidiurnal; S_2, the principal solar semidiurnal; N_2, the larger lunar elliptic semidiurnal; and K_1 and O_1 the lunar diurnal components. As an example showing how these constants may be used to determine, approximately, the principal characteristics of the tide, we may take Atlantic City where the values of the constants are as follows, the amplitudes being expressed in feet: M_2=1.91, $M_2°$=207°; S_2=0.39, $S_2°$=231°; N_2=0.45, $N_2°$=190°; K_1=0.34, $K_1°$=107°; O_1=0.25, $O_1°$=91°.

The M_2 tide represents the lunar semidiurnal tide and is thus the principal component of the tide. Its amplitude therefore is very nearly half the average range of the tide. In fact, it has been found that it generally represents about 98 per cent of the average half range; so that if we multiply the amplitude of the M_2 tide by 1.02 we will get approximately the half range. For Atlantic City, therefore, the range of the tide should be $1.02 \times 1.91 \times 2 = 3.9$ ft. From 9 years of continuous observations the range of the tide at Atlantic City is found to be 4.0 feet. The epoch of the M_2 tide should likewise represent very nearly the high-water lunitidal interval, that is, the interval between the moon's meridian passage and the first high water. To express the epoch in time it is merely necessary to divide the epoch by 360° and multiply by the period of the simple tide expressed in hours. For Atlantic City we therefore have $(207 \div 360) \times 12^h\ 25^m = 7^h\ 08^m$, which compares with $7^h\ 13^m$ derived from 9 years of observations.

Since the spring range of the tide occurs when the high waters of the solar and lunar tides combine and the neap range when the low water of the solar tide coincides with the high water of the lunar tide, we have spring range $= 2(M_2 + S_2)$, and neap range $= 2(M_2 - S_2)$. And on substituting the values of these components for Atlantic City we get 4.6 feet and 3.0 feet, respectively, which compare

with 4.8 feet and 3.1 feet from more accurate formulas. The perigean and the apogean ranges will likewise be given approximately by $2(M_2+N_2)$ and $2(M_2-N_2)$, which, upon substitution, give for Atlantic City 4.7 feet and 2.9 feet, respectively. The more accurate figures are 4.9 and 3.3.

It will be recalled that spring tides come about a day after full or new moon and that perigean tides follow the moon's perigee by about a day. These intervals vary from place to place and are known respectively as the phase age and the parallax age of the tides. The harmonic constants permit us to determine these ages very readily. It can be shown that the phase age in hours is given very closely by $S_2°-M_2°$ and the parallax age by $2(M_2°-N_2°)$. For Atlantic City we therefore get, by substitution in these formulas, phase age=24 hours and parallax age=34 hours. At Atlantic City, therefore, the spring tides occur one day after full or new moon, and perigean tides 1½ days after the moon's perigee.

In our study of types of tide we found that the diurnal inequality was the determining feature in the type of tide, and that the amount of diurnal inequality depended on the ratio between the daily and semidaily constituents of the tide. The daily constituent consists principally of K_1 and O_1 and the semidaily principally of M_2. Hence the ratio of K_1+O_1 to M_2 is a measure of the amount of the inequality in the tide at any place. Where this ratio is less than one-half, the tide is of the semidiurnal type; where it is between one-half and two, the tide is of the mixed type, and where greater than two the tide begins to be of the daily type. At Atlantic City the ratio of K_1+O_1 to M_2 is $(0.34+0.25)\div 1.91=0.31$, and hence the tide here is of the semidaily type.

The harmonic constants thus furnish a very satisfactory method of specifying the characteristics of the tide at any

point. But the harmonic analysis has still another advantage in that it furnishes a simple and efficient method for the prediction of the tide. This method, known as the harmonic method, is applicable everywhere and may be used with equal facility for all types of tide.

CHAPTER XIV

HARMONIC PREDICTION OF TIDES

THE harmonic analysis conceives the observed rise and fall of the tide at any point as being compounded of the rise and fall of a number of simple tides. But it also goes further than this in that it furnishes us with a method for determining accurately the amplitudes and epochs of these simple tides, or, in other words, their characteristics as regards time and height. The rise and fall of any simple tide may then be represented by a simple cosine curve, the amplitude determining the extent of rise and fall and the epoch determining the time of high water after the meridian passage of the hypothetical tide-producing body giving rise to that simple tide.

Suppose that for any given port for which harmonic constants are at hand we draw a cosine curve for each of the simple tides. A single complete period of each of these tide curves represents for all time the rise and fall of the corresponding simple tides with respect to the meridian passage of the respective hypothetical tide-producing bodies. To determine the height of any of these simple tides at any desired time of a given date, either in the past or in the future, it is only necessary to know the time of meridian passage for that day of the particular hypothetical tide-producing body. And in passing it may be noted that these times of transit for each of the hypothetical tide-producing bodies are tabulated for many years in advance in various tidal manuals. Clearly, then, by

adding the heights of the simple tides for any desired time, we may determine the height of the actual tide for as long a time in the future as we please. In other words, by the use of the simple cosine curves representing the rise and fall of the simple constituent tides, we may predict the tide for any desired time.

To make this matter clearer, let us predict by this method the times and heights of high and low water for the first day of January, 1926, at Sandy Hook, New Jersey. For the sake of illustration we need use but the five principal component tides discussed in the previous chapter, M_2, S_2, N_2, K_1, and O_1. From harmonic analysis of tidal observations at Sandy Hook, these constants have the following values, the amplitudes being expressed in feet: M_2=2.219, $M_2°$=218°; S_2=0.426, $S_2°$=246°; N_2=0.503, $N_2°$=201°; K_1=0.333, $K_1°$=102°; O_1=0.172, $O_1°$=98°. From a tidal manual we find that the hypothetical tide-producing bodies on January 1, 1926, will cross the meridian of Sandy Hook as follows: M_2 at 1:42 A.M., S_2 at 12:04 A.M., N_2 at 7:43 A.M.; K_1 will, at midnight, be one minute past its meridian passage, and O_1 2^h 12^m past its meridian passage. From the above data, together with the periods of revolution of these hypothetical tide-producing bodies, it is now a relatively simple matter to construct the curves for each of these components for January 1, 1926, as shown in Figure 45.

The rise and fall of the simple component tides is shown by the lighter curves, while the resultant rise and fall, compounded of the simple constituent tides, is shown by the heavy full-line curve. The height scale used is the same for all the curves and is shown in feet to the left. The curve for the M_2 tide, shown by the light full-line curve, brings out clearly the predominance of this constituent in the Sandy Hook tides. If now we read the times of high and low water of our resultant tide, we get as the

predicted times for January 1, 1926, low water at 2:55 A.M. and 3:35 P.M., high water at 9:05 A.M. and 9:40 P.M. The tide-table values, based on 23 component tides, are for this day as follows: Low water at 2:56 A.M. and 3:35 P.M.; high water 9:00 A.M. and 9:31 P.M. Our predicted times based on the 5 principal constituent tides agree very closely with the more accurate descriptions.

Reading now the heights of our resultant tide in Figure

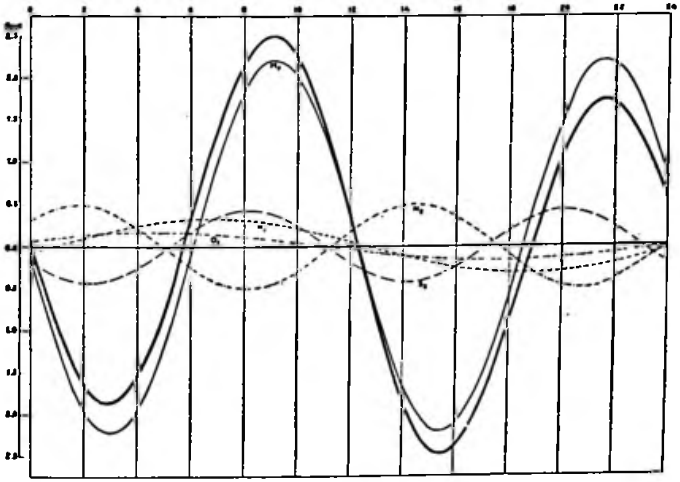


FIG. 45.—COMPONENT AND PREDICTED TIDE CURVES FOR SANDY HOOK, JANUARY 1, 1926

45, we find the low waters respectively 1.85 feet and 2.50 feet below mean sea level, and the high waters 2.50 feet and 1.75 feet above mean sea level. The predicted heights, however, are generally referred to the plane of mean low water which at Sandy Hook is 2.35 feet below mean sea level. To refer the heights of our predicted tides to the plane of mean low water it is therefore necessary to subtract our low-water heights from 2.35 and to add 2.35 to our high-

water heights. This gives for the low waters 0.50 foot and —0.15 foot, and for the high waters 4.85 feet and 4.10 feet. The tide-table values are respectively 0.0 foot and —0.4 foot for the low waters, and 4.7 feet and 3.8 feet for the high waters. Our predicted heights based on the 5 principal components are thus likewise in fairly close agreement with the more accurate predictions based on 23 component tides.

Theoretically, the method of harmonic prediction outlined above is very simple. The labor involved, however, is so enormous as to make it prohibitive for the practical purposes of tide tables. To predict with the desired degree of accuracy the time and height of the tide at any point, about 20 component tides are required. And since in a year there are approximately 1,400 high and low waters, it is obvious that an enormous amount of labor would be involved in predicting by this method the times and heights of the tide even for one port. Clearly, then, if we are to use the harmonic constants for the prediction of tides for practical purposes, some means must be devised for shortening very considerably the operations involved in the simple method illustrated. It is to the genius of Lord Kelvin that we are indebted for the practical solution of this problem. After devising the method of harmonic analysis in 1867, he put the harmonic method of prediction on a practical basis in 1872 by inventing a machine which mechanically effects the summation of the various component tides.

To understand the principles underlying the operation of the Kelvin Tide Predictor, let us first consider how we may produce a cosine curve mechanically. Various methods may be used, a simple one being illustrated diagrammatically in Figure 46.

In Figure 46, A is a crank revolving around a fixed center, C; on crank, A, is a pin, B, which can be set at

any desired distance from C. When A revolves, it moves this pin within a slotted slide, D, which has fixed to it a vertical rod carrying a pulley, E. As pin, B, moves in D, it causes pulley, E, to move up and down within the stationary frame, K, K. This movement of the pulley is transferred by means of a flexible wire passing over pulley, E, one end of this wire being fastened to a fixed point at F, and the other end passing over a fixed pulley, G, and

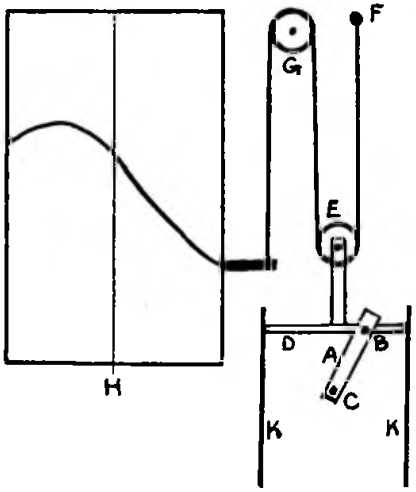


FIG. 46.—MECHANISM FOR DRAWING A COSINE CURVE

terminating in a pencil which touches a vertical drum that revolves on an axis, H. If now crank, A, and the drum are put in motion, the pencil will describe a cosine curve on the drum, as shown in the figure. This curve will have an amplitude equal to the distance from C to B, an epoch that is determined by the initial angle at which crank, A, is set, and a period determined by the relative motions of crank and drum. By varying these three factors we may

draw simple cosine curves of any desired amplitude, period, and epoch.

Suppose that instead of one crank-and-pulley system, as shown in Figure 46, we have several, each of which is set to draw a different cosine curve—different in period, in amplitude, and in epoch. If, now, we pass the wire in turn over each of the pulleys and set the system in motion, it is obvious that the curve drawn on the drum by the pencil will be a compound curve, compounded of the simple cosine curves which each of the various crank-and-pulley systems would have drawn if moving alone. In other words, by this means, the simple cosine curves are added together instantaneously and the resulting compound curve drawn on the drum.

Here, then, we have the principle underlying the construction and operation of the Kelvin Tide Predictor. For each of the simple constituent tides there is on the tide predictor a crank-and-pulley system, the crank having a movement relative to the drum which is determined once for all by the period of that component tide. When it is desired to predict the tide for any given port, each pin is set out a distance representing the amplitude of the particular component tide, and each crank is set at an angle representing the epoch of the tide. The machine is then put in motion, a roll of paper fed over the drum, and the curve of the tide drawn on this paper. When the tidal curve for a whole year has been made it is taken off the machine and the times and heights scaled off and tabulated for the tide tables.

The tide predictor may be run by clockwork or electric motor, and the curve run off in a few hours. Although the careful reading of the times and heights of the tide from this curve takes several days, this method of harmonic prediction is obviously a great advance over the prediction by the nonharmonic method, for, no matter how complicated

the tide, the tide predictor will give the curve with ease. Moreover, instead of complicated tables of corrections varying with the different types of tides, which the non-harmonic method necessitated, the harmonic method offers a simple and uniform procedure applicable alike to all types of tide.

As with all mechanical contrivances, the Kelvin tide predictor furnished but a model on which the ingenious minds of later generations were to work improvements. The scaling of the times and heights of the tide from the curve traced by the Kelvin tide predictor has several drawbacks. In the first place it is somewhat time consuming, several days being required to scale the times and heights of the tide over the period of a year for a single port. And in the second place it is difficult to read off these times to the nearest minute and heights to the nearest tenth of a foot. The device had the further drawback, too, that scaling times and heights from a curve is an operation requiring considerable care if errors are to be avoided.

To William Ferrel, a tidal mathematician of the U. S. Coast and Geodetic Survey, it occurred that a tide-predicting machine could be made which would indicate the time and height of the tide directly on dials. On his plans an ingenious tide predictor was constructed which indicated on dials on the face of the machine the times and heights of the predicted tide, and so did away with the curve traced on paper and the necessity for the scaling of the times and heights from this curve. The operation of the Ferrel machine consisted in turning a handle until two hands moving over the face of the machine came together. This was a signal to the operator that a high or low water was being indicated. He thereupon stopped the machine and read from the dials the time of the tide and also the height.

The Ferrel tide predictor was completed in 1882. Like the Kelvin machine it made use of the harmonic constants, but not directly as they result from the harmonic analysis. It is, therefore, in theory, somewhat more complicated than the Kelvin predictor. It was designed principally for the tides of the semidaily type, and for these gave excellent results. For the more complicated tides of the mixed and daily types, however, it had to be set twice, the first setting to give the time of the tide and the second setting to give the height of the tide. For such tides, too, it necessitated very careful manipulation on the part of the operator. And while the elimination of the tide curve reduced the time required for prediction, there were certain secondary purposes which the tide curve served to great advantage, so that its elimination was not an unmixed good.

In 1894, Dr. R. A. Harris, another tidal mathematician of the U. S. Coast and Geodetic Survey, called attention to the possibility of constructing a tide predictor that would combine the desirable features of both the Kelvin and Ferrel machines. This machine was to carry on, simultaneously, four operations; namely, (1) Draw the curve of the predicted tide, (2) Mark on the axis of the curve the times of each high and low water, (3) Exhibit on dials fixed on the face of the machine the times and heights of high and low water, and (4) Exhibit upon these dials the height of the tide at any desired time. This machine was later constructed in the instrument shop of the Survey, Dr. Harris being responsible for the mathematical principles involved, and E. G. Fischer, the mechanical engineer of the Survey, being responsible for the solution of the mechanical problems involved. The machine was completed in 1910, since which time it has been used for the prediction of the tide and current tables issued annually in advance by the Survey.

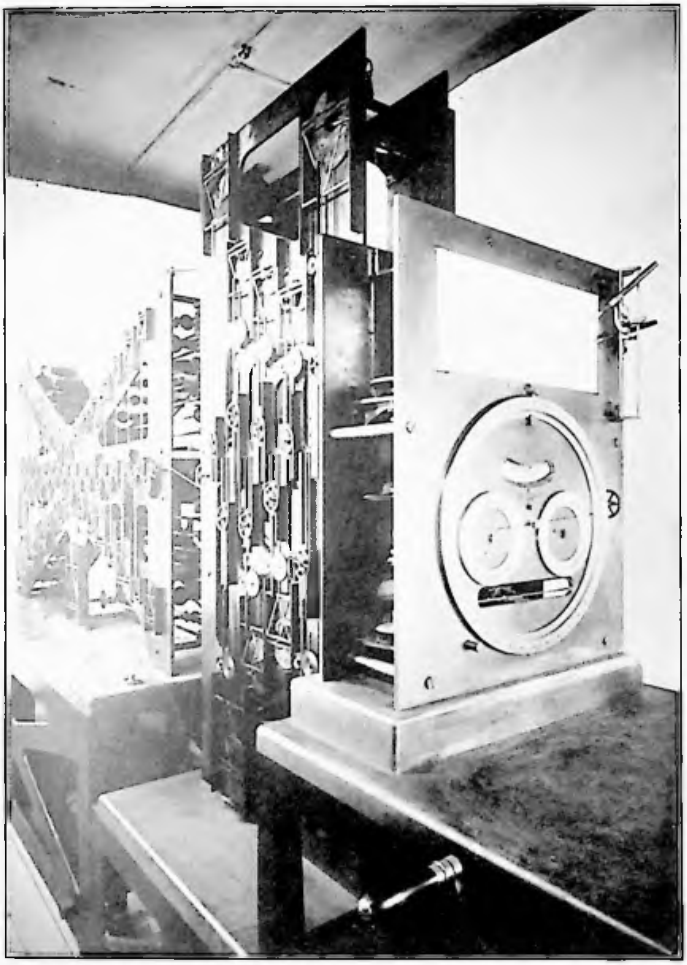


FIG. 47. U. S. COAST AND GEODETIC SURVEY TIDE PREDICTOR

A front view of the Harris-Fischer tide predictor is shown in Figure 47. In operation it is very simple. After the crank-and-pulley systems are set with the harmonic constants for a given port, the operator, who is seated facing the dials, turns the handle shown at the lower left of the figure and as each successive high and low water is reached the machine automatically stops. The two smaller dials then show the time of the tide, the one to the left the hour and the one to the right the minutes, while the larger dial shows the height of the tide. The operator records this time and height on a sheet of paper before him, releases the machine by a slight sidewise pressure on the handle and continues until the next tide is reached, when the machine again automatically stops and another record is made.

At the same time there is also being traced a curve of the predicted tide on a roll of paper that passes across the front of the machine above the dials. In Figure 47 two pens may be seen to the right above the dials. One of these pens traces the curve of the tide while the other draws a straight line on which it marks the beginning of each hour by a little downward jog, differentiating the midnight hour of each day by a double jog. This pen also indicates the times of each high and low water by an upward jog. The curve thus furnishes a complete record of the predicted tide.

The principle on which the tracing of the tide curve is based has already been considered, and in the Harris-Fischer machine the curve is obtained in a manner similar to that in the Kelvin predictor. The marking of the hours is a simple mechanical problem, and so is the indicating of the height of the tide by a dial. What is new is the indicating of the exact instant of high and low water and to understand how this is done we will have to consider a mathematical property of cosine curves.

For any given cosine curve there is another cosine curve known as the first derivative which has the same period as the original cosine curve but an epoch 90° less and an amplitude that has a fixed relation to the amplitude of the original curve. And it is the peculiar property of this first derivative cosine curve that its height is zero when the height of the original cosine curve is at its maximum or minimum, that is, when it is at a high or low water. Hence, if a tide predictor is made which sums on one side the simple cosine curves and on the other side the first derivative curves of these simple cosine curves, whenever the sum of these first derivative curves is zero, at that instant the tide is at high or low water. And it is this principle that the Harris-Fischer machine makes use of.

Now it is important to note that when a machine like the Kelvin predictor is drawing the curve of the predicted tide, we cannot determine the time or the height of high or low water at the instant it is being traced, for we cannot tell when the highest point is reached until that point is passed. But with any curve we can determine the instant when it reaches its mean level or zero height. On the Harris-Fischer predictor, one side of the machine is set with the amplitudes and epochs of the simple constituent tide curves, while the other side is set with the amplitudes and epochs of the first derivative tide curves. In all respects the crank-and-pulley systems of the two sides are alike. The wire passing over the pulleys set with the original harmonic constants is connected with the pen drawing the curve and with the large dial showing the height of the tide. The wire passing over the pulleys set with the first derivative harmonic constants merely winds and unwinds on a drum. But on this wire a platinum tip is fixed in an upright position; and every time it passes its mean level or the zero of its scale, it comes in contact with

another metallic point which closes the circuit of an electric current that locks the machine—indicating to the operator a high or low water, the time and height of which he reads off from the dial.

The Harris-Fischer tide predictor is arranged to take care of 37 simple tides, but for most ports from 20 to 30 are sufficient to give results with the desired degree of accuracy. To set the machine for any given port and make the necessary checks to insure freedom from error takes about 3 hours. After this setting the times and heights of the high and low waters for a whole year are predicted in about 7 hours and the predictions are then ready to be sent to the printer.

How accurately does the tide-predicting machine do its work? At first thought it would appear that a comparison of the predicted times and heights for a given port with the times and heights of the tide as they actually occur at that port would be a proper method of testing the accuracy of a tide predictor. But a moment's reflection will show that this is not the case. For what a tide predictor does is to sum up instantaneously the heights of a number of simple cosine curves, and the accuracy of the machine is to be judged by the precision with which it does this summing. When we compare the predictions with the tides as they actually occur we are only indirectly testing the accuracy of the tide predictor—what we are really testing in such a case is the accuracy of the mathematical processes of the harmonic analysis. If the theory and practice of harmonic analysis are not correct, then no matter how accurately the tide-predicting machine does its work, the predictions will not agree with the tides as they actually occur.

Let us therefore see how accurately the tide-predicting machine functions by comparing its results with the results of direct computation. Prior to its use for the prediction of tides for the tide tables, the Harris-Fischer

tide predictor was carefully tested by comparing the 24 hourly heights of a day as given by the machine and as determined by accurate computation. At Hongkong, China, the tide is of a somewhat complicated nature, although not of great range, and this station was used for the test. In 1910 the machine was set with 30 components for the prediction of the tides at Hongkong for the year 1912, beginning with the first of January. The machine was then run through to December 31 and for that day the height of the tide at the beginning of each hour was read off. December 31 was chosen as the day for comparison, since, being the last day of the year, any inaccuracies in the machine would presumably be shown to the fullest extent.

The height of the tide for each hour on December 31, 1912, for Hongkong was computed accurately "by hand," using the same components as were set on the tide-predicting machine. This involved a very considerable amount of laborious computation and required several days. It should be mentioned here that after the machine was set, the height of the tide for each hour on December 31 was read off in less than ten minutes. When the results were compared, it was found that the greatest difference between the machine reading and the accurate but laborious "hand" computation was six-hundredths of a foot. And it is to be remembered that for the use of the tide tables, the heights of the tide are given to the nearest tenth of a foot. For practical purposes, therefore, the tide-predicting machine gives results of high accuracy.

In 1922, after the Harris-Fischer machine had been in use for 12 years, it was again tested to determine whether or not its results were still sufficiently accurate. There are several thousand parts to the machine, a considerable number of these being moving parts, and there might therefore be some appreciable wear and tear. The pre-

dictor was again set with 30 components for Hongkong for January 1, 1912, and run through to December 31 of the same year, for which day the height of the tide for each hour was read. These hourly heights were compared with those predicted in 1910 and the greatest difference between the two sets of figures proved to be two hundredths of a foot.

Granting now that the tide-predicting machine performs its task with sufficient accuracy for all practical purposes, we are still confronted by the question as to how accurate the predicted tides are; that is, how closely do the predicted tides agree with the tides as they actually occur? Obviously, this is a relatively simple matter to determine, for we need merely compare the predictions for any given port with the tides as actually observed at that port. And it is to be observed that this comparison will test the accuracy of all the processes that enter into the prediction of tides.

The times and heights of the tide are subject to the disturbing influences of prevailing meteorological conditions. Heavy winds and sudden changes in atmospheric pressure will affect both the time and height of the tide; and until the meteorologist is in a position to predict a year or more in advance the exact meteorological conditions that will prevail, the predicted tides in the tide tables cannot take account of such changes. To test the accuracy of tidal predictions, therefore, we must choose a period that is as free as possible from the disturbing effects of wind and weather.

The accuracy of tidal predictions has been tested at different times for various ports. A recent test of the predictions as derived from the Harris-Fischer machine was made in 1920 by comparing the observations and predictions at two ports, one on the Atlantic coast, the other on the Pacific. These two ports were Portland, Maine, and

Seattle, Washington. The periods chosen for comparison were the months of May and November, 1919. The predictions used were made in 1917 in the ordinary course of preparing the tide tables for the use of the mariner. At Portland, the tide is of the semidaily type and has a rise and fall varying from 7 to 13 feet; at Seattle the tide is of the mixed type, the rise and fall varying from less than 2 feet to more than 16 feet.

At Portland, during the month of May, 1919, there were 119 high and low waters. The greatest difference between the predicted and the observed times of tide was 24 minutes, this difference occurring once during the month. Six times during the month there was a difference of as much as 18 minutes; 42 times the difference was 12 minutes, 52 times it was 6 minutes, and 18 times there was no difference at all. Putting the matter on a percentage basis, the results may be expressed as follows: For the predicted tides at Portland for the month of May, 1919, less than 1 per cent differed as much as 24 minutes, while 94 per cent differed by 12 minutes or less. For the month of November, 1919, again less than 1 per cent of the predicted tides at Portland differed by as much as 24 minutes from the observed tides; less than 3 per cent differed by 18 minutes, and 97 per cent differed by 12 minutes or less.

For Seattle the results were similar. Combining the results for the months of May and November, 1919, gives the following: Less than 1 per cent of the predicted tides differed by as much as 24 minutes from the observed tides; 21 per cent differed by 18 minutes, and 78 per cent by 12 minutes or less. It should perhaps be mentioned that in tabulating the tide from an automatic tide-gauge record, it is customary to read the times of high and low water to the nearest tenth of an hour. This explains why the differences between the predicted and the observed

times of tide as given above are in tenths of hours, that is 6, 12, 18, and 24 minutes.

It is to be noted that although no heavy storms occurred at the two stations during the months for which the comparison of predicted and observed times of tide were made, it is reasonable to suppose that part of the differences found may be fairly ascribed to the varying meteorological conditions. And it is to be remembered, too, that near the times of high and low water the height of the tide is changing very slowly. A difference between the predicted and observed times of tide of as much as two-tenths or even three-tenths of an hour (12 or 18 minutes) is generally of no consequence for practical purposes. We may therefore conclude from the above comparisons that the times of tide as given by the tide predictor are, for practical purposes, all that can possibly be desired.

When we come to the comparison between the heights of the predicted and observed tides, the agreement is not as close as in the case of the time of tide. And in this connection it is to be recalled that in the chapter dealing with the disturbing effects of wind and weather we found that these disturbances are generally more pronounced in the height of the tide than in the time. At Portland, for the month of May, 1919, the greatest difference between predicted and observed tides was 0.9 of a foot, which occurred once. Eleven per cent of the predicted tides differed between half a foot and 0.9 of a foot, while 89 per cent differed by half a foot or less. In November, wind and weather were more variable than in May, and this is reflected in the differences between the predicted and observed tides. On November 5, 6, and 7, 1919, there were high winds and considerable variation in barometric pressures at Portland, and for those three days the differences between predictions and observations are considerable, the greatest being 1.9 feet. But even if we include these days,

which obviously is unfavorable to the accuracy of predictions, we find that during November, 1919, at Portland 70 per cent of the predicted tides differed by half a foot or less from the tides actually observed; while 24 per cent differed between half a foot and a foot, and 6 per cent differed by more than 1 foot but less than 2 feet.

For Seattle, the comparison of the predicted with the observed heights of the tide for May, 1919, is as follows: 87 per cent of the predicted heights differed by half a foot or less from the tide as actually observed, and 13 per cent differed between half a foot and a foot, the greatest difference being one foot, which occurred only twice. For November, 1919, 78 per cent of the predicted tides differed by half a foot or less from the observed tides; 22 per cent differed between half a foot and one foot, the greatest difference between the predicted and observed tides for the month being one foot, and this occurred but once.

It is to be observed that Portland and Seattle do not afford as good a showing for the accuracy of the tidal predictions as other stations that might be chosen. Had San Diego, California, been taken, a comparison between predicted and observed tides for the month of May, 1919, would have shown 0.3 foot as the greatest difference, and 93 per cent of the tides within 0.2 foot. Summing up, therefore, we may say that in view of the disturbing effects of wind and weather, the prediction of tides by means of tide-predicting machines has now reached a high degree of accuracy. The close agreement between predicted and observed tides goes to prove, too, that the theory of the tides on which the harmonic analysis is based is a correct theory, and that the mathematical processes involved in the harmonic analysis and in the prediction of tides are accurate.

Before leaving the matter of harmonic prediction mention should be made of the fact that tidal currents lend

themselves to harmonic prediction in precisely the same way as tides. In fact, the current tables issued by the United States Coast and Geodetic Survey are predicted by the use of the tide-predicting machine.

CHAPTER XV

TIDAL ENERGY AND ITS UTILIZATION

In the rise and fall of the tide we have enormous quantities of water in motion, and moving water means energy. It is therefore small wonder that, in view of the vast extent of the ocean, people of an inventive turn of mind should direct their attention to the utilization of this seemingly inexhaustible source of energy.

In speaking of the utilization of the energy of the tides, we must be careful to distinguish between tidal energy and wave energy. The surface of the sea is never free from waves and the energy contained in these waves is sometimes utilized through the medium of wave motors. These waves, however, are not due to the tides and therefore their energy is not tidal energy. Strictly, tidal energy is the energy due to the raising and lowering of the level of the sea which is brought about by the tides.

In a very small way the energy of the tide has been utilized at various places for many years. A tide mill on the River Tamar in England, which was installed in 1790, is reported as still in operation. But it is only since the development, in recent years, of large hydroelectric power stations that large-scale projects for harnessing the tides have been seriously proposed. In Europe careful consideration has recently been given the matter. The French government has provided funds for the construction of a tidal power station of an experimental type near Brest; and in England consideration has been given to tidal power schemes involving the estuary of the Severn.

The practical utilization of the energy of the tides on a large scale centers about problems that are wholly of an engineering character, and therefore outside the purview of this volume. Here we want merely to see just what the possibilities of tidal power are and to consider briefly some of the means that have been suggested for the harnessing of this energy.

Recently an English engineer, Norman Davey, has published a book dealing with the utilization of tidal energy entitled *Studies in Tidal Power*. This is so clearly written that we can do no better than summarize some of its pages. Davey divides the various schemes that have been suggested for the utilization of tidal energy into four classes or systems which he calls, respectively, the float system, the tidal-stream system, the compressed-air system, and the basin system. Let us consider each of these briefly.

The float system is a very simple one. A floating body of considerable weight is lifted by the rising tide, and on the falling tide this body is made to do work by means of an appropriate system of levers and gears. It is not difficult to compute the power that is theoretically available through a system of this kind. Suppose we have a hulk weighing 1,000 tons floating in a tidal waterway having a range of 20 feet. On its fall this hulk will be capable of doing work represented by $2000 \times 1000 \times 20 = 40{,}000{,}000$ foot-pounds. This work is accomplished in a tidal cycle of 12½ hours; hence, the horse power is represented by $40{,}000{,}000 \div (12.5 \times 60 \times 33{,}000) = 1.62$. Altogether apart, therefore, from the mechanical difficulties of transmitting the slow motion of the falling hulk, the relatively insignificant amount of power realized by this scheme proves it to be of no practical value.

In the tidal-stream system use is made of one or more paddle wheels fixed on a raft anchored in the tidal waterway. As the tidal current floods and ebbs, the moving

water causes the paddle wheels to rotate. Without going into the details of calculation it may be said that while this system is more efficient than the float system it is applicable only to very small power units.

The compressed-air system makes use of the energy of the tide in the direct compression of air in closed chambers of suitable design. Calculation shows that while this system gives better results than the float or tidal-stream systems, it still gives but a minimum of power output and, considering the cost involved, is far too small to make the system at all practical. It appears, however, that use might possibly be made of this system for the direct compression of air for sirens.

The basin system—the last of the four systems—is the only one that presents the possibility of the practical utilization of the energy of the tides, in that it permits of large-scale installations. In principle it is simple, consisting essentially of one or more basins cut off from the sea by dams. As the tide rises and falls, suitable gates in the dams permit the maintenance of differences in level of water between basin and sea or between one basin and another during part of the tidal cycle. These differences in level produce the necessary heads of water for the production of power through turbines and auxiliary machinery.

How much power is theoretically available per unit area of the basin system? The calculation is not difficult, and to make this calculation applicable for various ranges and different areas of basins, let us take the range of the tide as R feet, and the area of the basin as A square miles. In passing, it is to be noted that there are 27,878,400 square feet in a square mile. Let us assume that at low water the gates are closed and remain closed during the entire period of succeeding rise. At the time of high water outside, there is a difference in head between the level of the water in the

basin and the water outside equal to R feet. If the gates now be opened, and if they be assumed large enough to permit the basin to be filled in a few minutes, a mass of water equal to 27,878,400AR cubic feet will have poured into the basin, the average distance of fall of the water being obviously ½R feet. And since the weight of a cubic foot of sea water is approximately 64 pounds, the amount of work that this mass of water is theoretically capable of doing is represented thus:

$$27{,}878{,}400 \times 64AR \times \tfrac{1}{2}R = 892{,}108{,}800AR^2 \text{ foot pounds.}$$

But this represents the work for the total rise or for a period of 6¼ hours. Hence the horse power theoretically available is $892{,}108{,}800AR^2 \div (6.25 \times 60 \times 33{,}000) = 72AR^2$.

The tidal power available therefore depends on two factors, namely, the area of the basin enclosed and the range of the tide. But it is important to note that while the power increases simply as the area of the basin increases, in the case of the range the increase is not simply as the increase in range but as the square of the range. In other words, as between two regions, one having a tidal range of 5 feet and the other a range of 20 feet, other things being equal, the tidal power available in the latter region is 16 times as great. It follows, therefore, that if tidal power ever becomes commercially practicable on a large scale, it will be first in regions of considerable range. And in this connection it is to be noted that the tidal-power schemes seriously considered at present, those of the Severn in England and near Brest in France, involve tidal waters in which the range is more than 20 feet in the one case and more than 15 feet in the other.

Our formula shows that in regions where the tide has a range of 10 feet, every square mile of tidal water impounded is theoretically capable of producing 7,200 horse power, while with a tidal range of 20 feet the horse power

available is 28,800. These are impressive figures, for it is not difficult to conceive basins with areas up to 20 square miles or more; which means, theoretically, 144,000 horse power in regions of 10 feet range and 576,000 horse power where the range is 20 feet. It should therefore not be overlooked that these figures represent the *maximum* horse power *theoretically* available. Actually only a fraction of this power can be obtained.

In arriving at our formula it was assumed that the difference in head of water between the sea and enclosed basin was equalized in a few minutes. Practically, of course, this is not the case, for the water going through turbines would require several hours to make use of the difference in head, and this means that advantage cannot be taken of the full range of the tide. Moreover, turbines cannot in practice utilize all the energy of a head of water varying from R feet to 0 feet. The theoretically maximum possible power is thus several times the power that can actually be realized.

There are certain inherent difficulties in the development of tidal power that must not be overlooked. The tidal day is 24 hours and 50 minutes in length. Each day, therefore, the times of high and low water come later, on the average, by 50 minutes. This introduces a somewhat troublesome factor, for the day on which mankind bases its activities is exactly 24 hours in length. Furthermore, since in connection with the rise and fall of the tide a head of water can be maintained only for part of the tidal cycle, tidal power is in its development of an essentially intermittent character.

Another difficulty in the utilization of the energy of the tide arises from the variation in range from day to day. This brings in its train two difficulties. In the first place, the variation in head of water, consequent on the variation in range, introduces difficulties in connection with the

proper functioning of the turbines. In the second place, the variation in range brings about considerable variation in the tidal power developed.

The most important variation in range is that within the period of a fortnight. This variation is different for different regions, but in general we may take the range of tide at times of new and full moon—when the so-called spring tides occur—as 50 per cent greater than at the times of the moon's first and third quarters when neap tides come. Since tidal power varies as the square of the range of tide, the power developed at spring tides is about 125 per cent greater than at neap tides. And during the year there are even greater variations, for at the times of perigean spring tides the range may be 2½ times as great as at the times of apogean neap tides, which means that the power that may be developed at the one time is about 525 per cent greater than at the other.

To overcome the difficulty of variability in power output from day to day, various plans have been suggested. One of these would make use of the excess power during spring tides in pumping water into a high-level storage reservoir which could then be used for the production of additional power in a secondary power plant during periods of neap tides. Another plan is to make use of an auxiliary coal or oil-burning power plant which would be operated during periods of low tidal-power development to equalize the power output from day to day.

A number of factors enter into the question of the practicability of tidal power development at any particular place. But in general it may be taken that where the average range of the tide is less than 10 feet the development of tidal power is not economically feasible. There are many localities where the range of the tide is 10 feet or more. But a search through the tide tables will show that there are not many regions where the range of the

tide is 20 feet or more. The table following lists the regions in which the range of the tide is as much as 20 feet.

1. Bay of Fundy, Canada, 44 feet.
2. Port Gallegos, Argentina, 36 feet.
3. Frobisher Bay, Davis Strait, 35 feet.
4. England, West Coast, 33 feet.
5. Koksoak River, Hudson Strait, 30 feet.
6. Cook Inlet, Alaska, 30 feet.
7. Magellan Strait, Chile, 30 feet.
8. France, North Coast, 28 feet.
9. Ashe Inlet, Hudson Bay, 23 feet.
10. Collier Bay, Western Australia, 23 feet.
11. Arabian Sea, India, 23 feet.
12. Maraca Island, Brazil, 23 feet.
13. Colorado River, Mexico, 22 feet.
14. Chemulpo, Korea, 21 feet.
15. St. Croix River, Maine, 20 feet.

The table above brings out one important fact, namely, that a number of the places which possess large tidal-power possibilities are situated near the Arctic regions or in localities remote from centers of population. In the final analysis the question of utilizing the energy of the tide is one of relative cost. At present it is not at all certain that, as an independent unit to supply power for ordinary industrial purposes, any large-scale tidal-power development would be financially successful. However, unless other sources of power are opened up by inventive genius, there can be little doubt that with the exhaustion of coal and oil, tidal power will become of great importance, especially in regions having a considerable range of tide. But the facts in the case do not lend color to the pleasant prophecies, at times heard, that in the not far-distant future mankind will derive the power demanded by a civilized world from the inexhaustible energy of the tide.

CHAPTER XVI

THE GREATEST TIDE IN THE WORLD

THE greatest known rise and fall of tide occurs in the Bay of Fundy, the arm of the Atlantic that separates the Canadian province of Nova Scotia from its sister province of New Brunswick and from the State of Maine. At its head, the Bay of Fundy forks into two inlets, in the southern one of which, Minas Basin, the tide rises a vertical distance of from 40 to 50 feet in a period of 6 hours and falls the same distance in the following period of 6 hours. In some of the older publications the figures given for this rise and fall appear to have been affected by a "factor of dramatic effect." As an example, we find in the sixth edition of Sir John Herschel's *Astronomy,* published in 1859, the statement that "in some places the tide-wave, rushing up a narrow channel, is suddenly raised to an extraordinary height. At Annapolis, for instance, in the Bay of Fundy, it is said to rise 120 feet." As given in recent tide tables the rise and fall of the tide at Annapolis is much more prosaic, being on the average 25 feet and increasing to 29 feet at times of spring tides.

In the upper parts of the Bay of Fundy, in consequence of the great range of the tide, the shores at low water present a bare appearance, exposing to view wide expanses of mud flats. This condition greatly affects communication by water. Small steamers in the upper part of the Bay have to make their calls at high water and leave promptly, as all the wharves are dry at about half tide. Schooners are accommodated by lying on a bench of mattress work,

against the wharves, while the tide is down. A typical Bay of Fundy wharf is illustrated in the frontispiece.

The configuration and hydrographic features of the Bay of Fundy are shown in Figure 48. The mouth of the Bay, defined by a line drawn from Jebogue Point on the south to Point of Main on the north, has a width of 87 miles and an average depth of 280 feet, reckoned from mean sea level. At Cape Chignecto, where the Bay forks, the width is 30 miles and the average depth 130 feet, and above Cape Chignecto the contraction in width and shallowing in depth take place even more rapidly. It is to this narrowing of the Bay and its lessening depth from mouth to head that the very considerable rise and fall of the tide in the upper reaches had generally been ascribed.

The amount of the rise and fall of the tide increases from the mouth of the Bay to its head. Taking the Nova Scotia shore, we have a mean range of 9.1 feet at Cape Sable, 14.0 feet at Yarmouth, 18.2 feet at Grand Passage, 24.1 feet at Digby Gut, 27.8 feet at Port George, 31.5 feet at Black Rock Light, 42.0 feet at Horton Bluff, and 44.2 feet in Noel Bay. At times of new and full moon, when the so-called spring tides occur, the range of the tide is about 14 per cent greater, and at such times the tide in Noel Bay has a rise and fall of about 50.5 feet.

On the north shore a similar condition prevails, the range of the tide increasing from the mouth of the Bay to its head. This increase becomes even more striking if we begin with the tide at Tom Nevers Head on Nantucket Island and go up the coasts of Massachusetts, New Hampshire, Maine, and New Brunswick, which form the western and northern shore of the Gulf of Maine and Bay of Fundy. For the mean range of the tide we have 1.2 feet at Tom Nevers Head, 3.7 feet at Monomy Point, 6.0 feet at Nauset Harbor, 8.9 feet at Gloucester, 10.2 feet at Bass Harbor, 15.7 feet at West Quoddy Head, 20.9 feet at St. John, 26.3

feet at Quaco, and 39.4 feet at Folly Point. Here, likewise, the spring range of the tide is about 14 per cent greater than the mean range.

A close examination of the ranges of the tide brings to light the fact that, relative to the axis of the Bay, a point on the southern shore has a greater range of tide than the corresponding point on the northern shore. Thus the

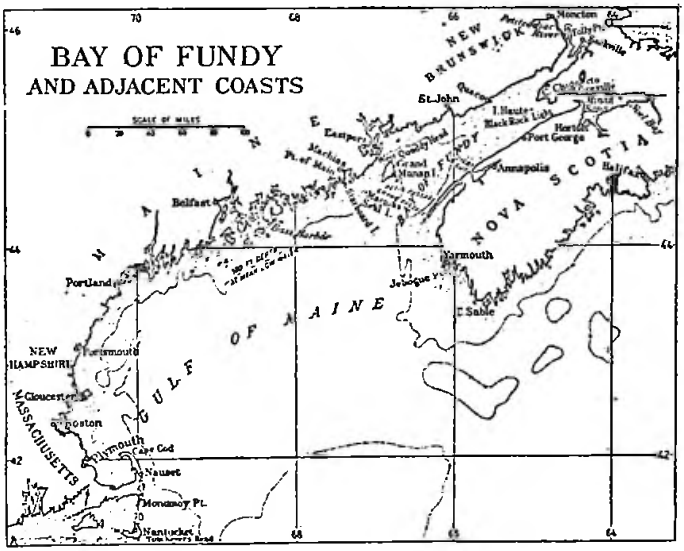


FIG. 48.—HYDROGRAPHIC FEATURES, BAY OF FUNDY

tide tables give as mean ranges of the tide 14.0 feet for Yarmouth against 12.9 feet at Starboard Island; 19.3 feet in Petit Passage against 15.7 feet at West Quoddy Head; 27.8 feet at Port George against 26.3 feet at Quaco.

This increase in the range of the tide on the southern shore is brought about by the rotation of the earth, in consequence of which all moving bodies are impressed

with a force deflecting them to the right in the northern hemisphere and to the left in the southern hemisphere. On the flood tide the water entering the Bay of Fundy is deflected to the right or southern shore; hence high water here is raised somewhat higher than on the northern shore. On the ebb tide the moving water is again deflected to the right; but now it is the northern shore that is to the right of the moving water, and hence low water is somewhat higher on the northern than the southern shore. The tide therefore rises higher and falls lower on the southern shore, giving a greater range here than on the northern shore.

In its rise and fall the tide in the Bay of Fundy exhibits the characteristic features of the semidaily tide of the north Atlantic coast. In general, the tide curve for any point of the Bay does not present, apart from the range of tide, any unusual features. Indeed, the form of these tide curves is unusually regular. But as we know, the tide in its rise and fall does not move at a uniform rate, the most rapid rise or fall occurring midway between low water and high water. With a tide having a range of 40 to 50 feet we may therefore expect at such times a very rapid change in the level of the water. In 1916, in the vicinity of Noel Bay, Dr. W. Bell Dawson measured a rise of 11.2 feet per hour occurring with a tide that had a total rise of 49.7 feet in a period of six hours.[1]

[1] It may be of interest to note here that in one of our encyclopedias, published about 65 years ago, there is the statement that the Bay of Fundy was "remarkable for its extraordinary tides which rush up from the sea with such rapidity as sometimes to overtake swine feeding on . . . the shores." And in this connection there is a story that reflects considerable credit on the intelligence of the swine. According to this story, before descending to the lower stretches of the shore laid bare by the falling tide, the swine of the neighborhood would always leave one of their number stationed on a high bluff looking toward the sea. On the approach of the rapid rise of water, this sentry would squeal a warning to the herd

To what is the extraordinary rise and fall of the tide in the Bay of Fundy due? It is evident that the effect of the converging shore lines and gradually decreasing depth is to increase the range of the tide in the upper reaches of the Bay by confining the energy of the moving mass of water into a smaller volume. But it is likewise evident that these features alone cannot be responsible for the increase in range from less than 10 feet at the mouth to more than 40 at the head. There is involved some factor other than the "cooping up of the tidal wave as it advances up the Bay." For in many other bays we have somewhat similar features without anything like corresponding increases in range of tide. Thus, at the mouth of Delaware Bay we have a mean tidal range of 4½ feet; and at Philadelphia, where the cross section of the tidal waterway has decreased very considerably, the range of the tide is but little over 5 feet.

If we examine the time relations of the tide in the Bay of Fundy we find that while the tide becomes somewhat later in going upstream, this retardation in time is quite inconsiderable. Thus from Machias Seal Island, near the mouth of the Bay, to Isle Haute about 90 miles upstream the difference in time is about 5 minutes. Furthermore, on investigating the relation of current to tide, it is found that throughout the Bay the slack of the current comes near the times of high and low water, while the strength of the current occurs midway between high and low water. These features, together with the increasing range of tide from mouth to head, prove conclusively that the tidal movement in the Bay of Fundy is primarily of the stationary-wave type. That is, the movement of the water in the bay is similar to that in Long Island Sound discussed in Chapter VIII, and which in diagrammatic form is repre-

feeding below, this warning giving them sufficient time to scamper to safety.

sented by the movement of the water in half the tank shown in Figure 30.

One reason for the increased range of the tide in the upper reaches of the Bay of Fundy is therefore found in the character of the tidal movement; for as shown in the discussion of the stationary-wave type of tidal movement, the rise and fall of the tide increases with the distance from the axis about which the tide oscillates, and in bodies of water opening at one end into larger basins this axis is located at the mouth of the body of water. A further reason is found in the greatly decreased cross section of the upper end of the Bay, which is brought about by the very considerable contraction in width and shallowing in depth. From a width of 87 miles at its mouth with an average depth of 280 feet, the Bay at its head becomes reduced to a few small shallow streams.

But even the coöperation of the two causes adduced above does not satisfactorily account for the enormous range of the tide found in the Bay of Fundy. A still further cause is found in the period of oscillation of the water in the Bay as a whole. Referring again to the discussion of the tidal movement in Long Island Sound (Chap. VIII), it will be recalled that bodies of water opening at one end into larger basins have natural periods of oscillation depending on the length and depth of the body, the formula for a rectangular tank of uniform depth being approximately $T=\frac{4L}{\sqrt{gh}}$, where T is the period of oscillation, L the length of the body, g the acceleration of gravity and h the depth of the water.

It is somewhat difficult to assign exact values to the length and average depth of a body of water like the Bay of Fundy. Approximately, however, we may take its length as 170 miles and its average depth as 240 feet. Taking the value of g as 32.17 feet per second, we have, from our formula above, expressing the units in feet and

seconds, $T=\frac{4\times170\times5280}{\sqrt{32.17\times240}}=40{,}860$ seconds, or 11.4 hours The natural period of oscillation of the Bay of Fundy is thus approximately 12 hours.

We may therefore conclude that the tidal phenomena in the Bay of Fundy are due primarily to the fact that the natural period of oscillation of the Bay approximates the period of the ocean tide. This brings about a stationary-wave movement within the Bay that is sustained by the tide of the open ocean which has approximately the same period. In the upper reaches of the Bay the range of tide is further increased because of the considerable diminution of cross section brought about by the contraction in width and shoaling of bottom.

Apart from the very large range, the tides in the Bay of Fundy present another striking phenomenon, and that is the bore, which is seen to best advantage at Moncton on the Petitcodiac River. This, however, has already been discussed in connection with river tides in Chapter VI and hence here we need only refer to it in passing.

The Bay of Fundy is a large body of water and over the whole of its area the rise and fall of the tide is considerable. Potentially, therefore, it represents a very large reservoir of power. And since the spell of large numbers is a potent one, it may not be out of place here to calculate just how much energy is represented by the rise and fall of the tide in the Bay of Fundy. For our purpose, however, approximate values will suffice, and therefore we may use round numbers.

Taking the mouth of the Bay to be defined by a line from Jebogue Point on the south to Point of Main on the north, the area of the Bay of Fundy is, in round numbers, 6,000 square miles. In round numbers, too, we may take the range of the tide as 13 feet at the mouth and as 40 feet at the head. If the Bay were rectangular in shape, the average range over its entire area could be taken as

half the sum of the ranges at mouth and head; but since its shores converge toward the head, the average range will be given more correctly by taking it as less than half but greater than one-third the sum of the ranges at mouth and head. Splitting the difference between one-half and one-third, we get five-twelfths, which gives the average range of the tide for the Bay as 22 feet. Substituting these values in the formula for the horse power available in a tidal waterway (derived in the preceding chapter), namely, $72AR^2$, where A is the area of the basin in square miles and R the range of the tide in feet, we get $72 \times 6{,}000 \times 22 \times 22 = 209{,}088{,}000$ horse power.

With 200,000,000 horse power to work on, an active imagination would not find it difficult to draw an alluring picture of the tidal-power possibilities of the Bay of Fundy. It is therefore to be emphasized again that such figures represent the maximum horse power theoretically available by impounding the whole of the Bay by a dam across its mouth which is nearly 90 miles long. By the time the scheme for utilizing the energy of the tide in the Bay of Fundy is reduced to practical details, the figures for the horse power actually available will unfortunately lose much of their impressiveness.

Having succumbed to the spell of numbers, we may go a step farther and calculate the amount of water that flows in and out the Bay of Fundy during a single flood or ebb tide. Here again round numbers will suffice for our purpose. Since the tide is very nearly simultaneous over the whole of the Bay, it follows that the quantity of water flowing into the Bay on a flood tide is very nearly equal to the area of the Bay multiplied by the average range of the tide. These factors we have found to be 6,000 square miles and 22 feet, respectively. Bearing in mind that a square mile contains $5{,}280 \times 5{,}280 = 27{,}878{,}400$ square feet, we have as the amount of water brought into the Bay by

the flood tide as $27,878,400 \times 6,000 \times 22 = 3,679,948,800,000$ cubic feet. It may help in visualizing the matter to state that in a period of 6 hours the tide brings into the Bay of Fundy, with an area of 6,000 square miles, very nearly as much water as, on the average, falls in the form of rain during a week over all of the United States, with an area of more than 3,000,000 square miles.

The enormous quantity of water flowing in and out the Bay of Fundy, together with the considerable range of the tide, might lead one to conclude that the tidal currents in the Bay are exceptionally strong—rivaling, perhaps, the exceptional rise and fall of the tide. It will be recalled that in the chapter devoted to the relation of current to tide it was shown that there is no such simple relationship between current and tide as would be expressed in the statement "large tides, strong currents; small tides, weak currents." We can calculate the velocity of the tidal current in the entrance to the Bay of Fundy by making use of the quantity of water calculated in the preceding paragraph, as is evident from the following considerations.

Since we know the total quantity of water flowing into the Bay during the flood period, it follows that, if the velocity of the tidal current were constant during the entire period of flood, this velocity would be equal to the volume of tidal water entering divided by the area of the cross section through which it enters. If we denote the volume of water by W, the area of the cross section in square feet by A, and the average velocity of the current in knots per hour by V_a, we have, remembering that the knot is a velocity of 6,080 feet per hour and that the duration of a flood or ebb period is 6.21 hours, $V_a = W \div (6.21 \times 6080A)$.

We have already determined the value of W, and the value of A is determined from the width of the mouth of the Bay and the average depth of the mouth. Taking the

mouth of the Bay, as above, from Jebogue Point to Point of Main, its length is 87 miles and its average depth is 280 feet, giving for the cross section here an area of $87 \times 5{,}280 \times 280 = 128{,}620{,}800$ square feet. Substituting now these values of W and A in the formula of the preceding paragraph, we have $V_a = 3{,}679{,}948{,}800{,}000 \div (6.21 \times 6{,}080 \times 128{,}620{,}800) = 0.76$ knots.

It is known, both from theory and from observation, that in a moving mass of water the velocity is greatest in the center of the channel. From numerous observations it has been found that the velocity in the center of a channel is about 1⅓ times as great as the average velocity. Hence, for the entrance to the Bay of Fundy the average velocity of the current in the center of the channel is $0.76 \times 1.33 = 1.01$ knots. To determine now the velocity of the tidal current at the time of strength from the average velocity, we make use of a mathematical property of the cosine curve, which is that its greatest height is 1.57 times its average height. For the velocity of the tidal current in the center of the entrance of the Bay of Fundy at strength we therefore derive $1.01 \times 1.57 = 1.59$ knots.

Is there any way in which we can check our calculations? For a velocity of 1½ knots is a very moderate velocity, and, in comparison with the range of tide found in the Bay of Fundy, relatively insignificant. It happens that we can test our calculations by observed fact, for in 1904 and again in 1907 the Canadian Tidal and Current Survey measured the velocity of the current at a number of points in the Bay of Fundy; and in the entrance to the Bay the observed velocities were found to be about 1½ knots at strength, which agrees very well with our calculated velocity.

CHAPTER XVII

FLOOD AND EBB IN NEW YORK HARBOR

UNLIKE other harbors which generally consist of a tidal bay or river, New York Harbor comprises a system of five tidal waterways that connect with the sea by means of two inlets many miles apart. Figure 49 shows the position of New York Harbor with regard to the ocean, and Figure 50, on a larger scale, shows the relations of the various parts of the harbor to each other. The principal entrance to the harbor is the southerly one past the Narrows, which is the gateway of important trade routes. Through the Narrows come the ships from all the ocean highways, and through this same funnel the exports of a great hinterland, reaching back of the Alleghenies, find their way into the channels of international trade. Past this gateway the largest ships afloat may steam into the harbor at any stage of the tide.

The Narrows leads into Upper Bay, a sheet of water having an area of about 15 square miles, and from this bay three tidal highways radiate. Toward the west, Kill van Kull, a short strait, connects with Newark Bay; toward the east, East River, a strait about 14 miles long, leads to Long Island Sound, the eastern entrance from the ocean to New York Harbor; and into the northern end flow the waters of the Hudson, a tidal river that drains a territory of some 13,000 square miles. Harlem River, the fifth of the waterways, is a strait about 7 miles long connecting East River with the Hudson. New York Harbor may

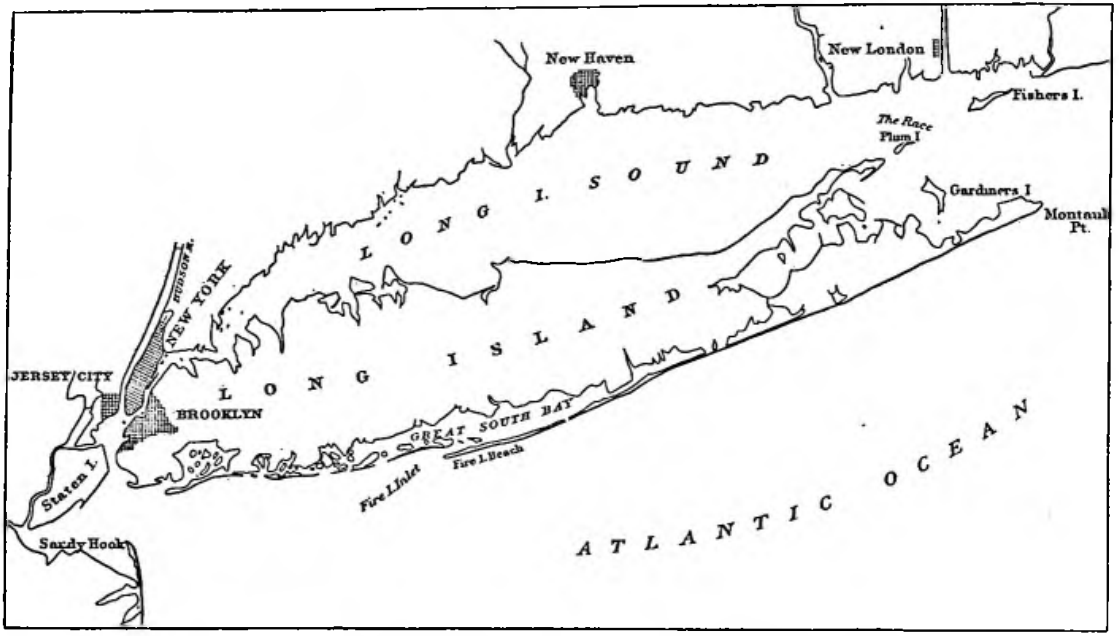


FIG. 40.—NEW YORK HARBOR, SHOWING THE TWO OCEAN ENTRANCES

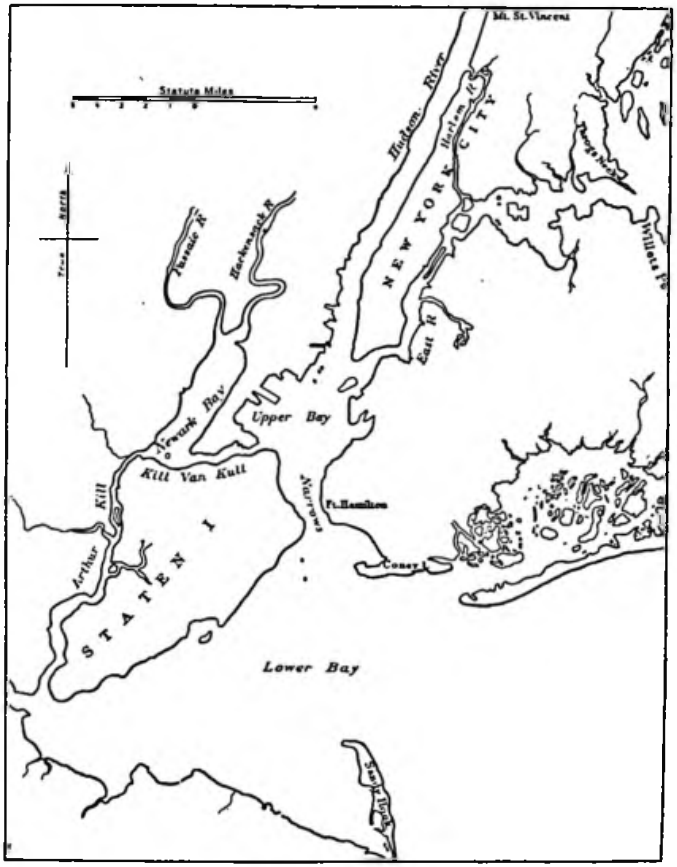


FIG. 50.—NEW YORK HARBOR. SHOWING THE RELATIONS OF THE COMPONENT WATERWAYS

therefore be taken as consisting of Upper Bay, Kill van Kull, East River, Harlem River, and Hudson River from its mouth to Spuyten Duyvil, where it meets the Harlem.

It is obvious that a case may be made out for including,

as parts of New York Harbor, Newark Bay and Arthur Kill on the west, Lower Bay on the south, and even Long Island Sound on the east. However, the five waterways enumerated above constitute New York Harbor as usually understood and for our purpose will be taken to comprise the harbor. This procedure may be justified, too, by the fact that an understanding of the tidal phenomena in New York Harbor, in the more restricted sense used here, may be arrived at without the need of discussing in detail the tidal régime in Newark Bay, Arthur Kill, Lower Bay, or Long Island Sound.

A glance at Figure 49 shows that the tide from the Atlantic Ocean reaches the harbor of New York from two directions. From the east the ocean tide enters East River after traversing Long Island Sound, while from the south the tide sweeps into the Narrows past Sandy Hook and Lower Bay. There is, to be sure, another channel for the tide, namely, from Lower Bay through Arthur Kill into Newark Bay and thence through Kill van Kull into Upper Bay. But the Narrows affords a much shorter and a very much more commodious channel, so that relatively very little of the ocean tide finds its way into the harbor by way of Arthur Kill. The tidal gateways to New York Harbor may therefore be said to be located at Throgs Neck on the east and at Fort Wadsworth on the south.

Into New York Harbor, therefore, the ocean tide finds its way through two inlets many miles apart. This in itself makes possible some variety in the tidal phenomena of the harbor; but still further variety is brought about by the fact that the five component parts which constitute the harbor are either intercommunicating or lead into other waters, so that the tide in one part may be affected by the tide in another.

The tides in the Narrows, the southern gateway to New York Harbor, are quite well determined. For more than a

quarter of a century an automatic tide gauge has been in operation at Fort Hamilton, furnishing a continuous record of the rise and fall of the water night and day, winter and summer. In its larger features, the tide in the Narrows is of a relatively uniform character from day to day; but a close examination shows certain variable characteristics. The varying positions of sun and moon in the heavens, the changing winds, unusual rainfall, freshets in the upper

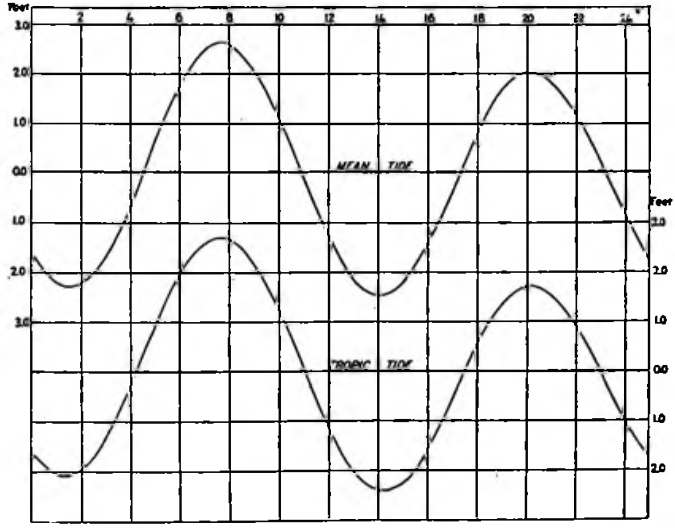


FIG. 51.—TIDE CURVES, FORT HAMILTON, NEW YORK

reaches of the Hudson, and even changes in barometric pressure—all these variations are reflected in the rise and fall of the tide.

The upper diagram of Figure 51 represents the average conditions of the rise and fall of the tide at Fort Hamilton, while the lower diagram represents the tropic tide, when the moon attains its maximum semimonthly declination.

A glance shows that the tide here is of the semidaily type, morning and afternoon tides being very much alike. When the moon crosses the meridian of Fort Hamilton the tide is falling, and the fall continues an hour and 38 minutes longer, at which time low water is reached. The tide then begins rising and 7 hours 40 minutes after the moon has crossed the meridian high water is reached. The average rise and fall of the tide is 4.7 feet, the rise taking place in $6^h\ 02^m$ and the fall in $6^h\ 23^m$.

It is to be noted that this diagram represents average tidal conditions. Since the sun, too, is concerned in bringing about the rise and fall of the water, the time of tide is not absolutely fixed with relation to the moon's meridian passage. If the lunar constituent could be restrained from interfering, the component of the tide which the sun brings about would produce high water at Fort Hamilton about 8:00 o'clock both morning and evening. This solar tide has at Fort Hamilton a rise and fall of but 1 foot; nevertheless it affects both the time and height of the lunar tide. The intervals by which high and low water follow the moon's meridian passage are therefore somewhat variable, though it is to be noted that at Fort Hamilton this variation is only infrequently as much as an hour.

The mean tidal curve shows that at Fort Hamilton the average difference in the heights of the two low waters of a day is four-tenths of a foot, while in the high waters this difference is six-tenths of a foot. When the moon has its greatest semimonthly north or south declination the diurnal inequality in the heights of high or low waters is at a maximum, the lower curve in Figure 51 showing the character of the tide at such times. It is seen that the low-water inequality has been changed but little, being at such times six-tenths of a foot; the high-water inequality, however, now becomes 1 foot. In other words, the diurnal inequality in the tide at Fort Hamilton—and also through-

out New York Harbor—exists principally in the high waters.

At times of full and new moon the tidal forces of sun and moon conspire and bring about a rise and fall of the tide greater than the average—the so-called "spring tides." At Fort Hamilton at such times the tide has a range 20 per cent greater than the mean range, the average rise and fall of the tide then being 5.6 feet. At times of the moon's first and third quarters, the tidal forces of sun and moon are opposed, neap tides then occurring. At such times the tide at Fort Hamilton has a range of 3.8 feet, or 20 per cent less than the mean range. The distance of the moon from the earth, as we have found, likewise affects the range of the tide. When the moon is in perigee—nearest the earth—the mean range of the tide at Fort Hamilton is increased by 23 per cent, becoming 5.8 feet; and when the moon is in apogee the range of the tide is decreased 20 per cent, being then only 3.8 feet.

It follows, therefore, that when the moon is full or new at the same time that she is in perigee, the tides at Fort Hamilton will be about 40 per cent greater than the average; and that at times when the moon is in her first or third quarters when she happens also to be in apogee the tides will be about 40 per cent less than the average. But it is to be noted that in its response to the variations in the position of the moon relative to the earth and sun, the tide at Fort Hamilton is about a day behind the moon; that is, the range of the tide is greatest, not on the day of full moon, but a day later. Likewise the range of the tide is greatest about one day after the moon is in perigee, and it is by an interval of a day that the neap tides follow the moon's first and third quarters. This lag in the response of the tide is known as the "age of the tide" and, as previously stated, is generally ascribed to the effects of friction.

In addition to the changes due to the celestial conditions noted above, the rise and fall of the tide at Fort Hamilton is also considerably influenced by terrestrial conditions. This is to be expected for two reasons. In the first place, the mobility of the water permits the wind to affect the time and height of the tide, and, in the second place, the Narrows acts as a funnel through which there is discharged into the sea the fresh-water run-off from a territory nearly 15,000 square miles in area. This fresh-water discharge varies throughout the year, depending on the variations in rainfall and also on freshet conditions in the Hudson River. And since the variations in wind and rainfall are largely of a seasonal nature, the tides at Fort Hamilton should likewise exhibit a seasonal variation. This is shown in Figure 52, which represents in diagrammatic form the average heights of high water and low water each month of the year as determined from observations extending over a period of 19 years, from 1902 to 1920.

The vertical lines represent the average monthly rise and fall of the tide at Fort Hamilton for each month of the year, the upper end of each of these vertical lines showing the average monthly height of high water, while the lower end shows the average monthly height of low water. The dotted horizontal line joining the upper ends of the vertical lines therefore represents the monthly changes in the height of high water, and the lower dotted horizontal line likewise represents the monthly changes in the height of low water. It is important to note that the seasonal variation in the height of the tide is the same for both high and low waters. In other words, this seasonal variation does not affect the range of the tide, but it does bring about a corresponding seasonal variation in sea level at Fort Hamilton, as shown by the dashed line.

Figure 52 brings out the fact that, so far as seasonal changes are concerned, high water and low water at Fort

Hamilton are lowest in the winter months with a minimum about the latter part of January and are highest in the summer months with a maximum in the early part of August. Sea level at Fort Hamilton therefore goes through the same annual change, the difference in the average height of sea level between the latter part of January and the early part of August being seven-tenths of a foot.

The monthly variations in the heights of the tide noted above are average variations; obviously the variations produced by any individual storm or freshet would be con-

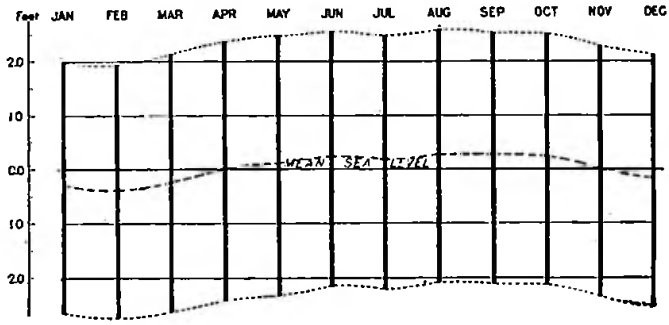


FIG. 52.—ANNUAL VARIATION IN HEIGHT OF TIDE, FORT HAMILTON, NEW YORK

siderably greater. On December 7, 1914, a northeast gale that had been blowing for almost two days, with a velocity at times exceeding 60 miles an hour, brought about a tide which at high water stood 5.1 feet above mean sea level—almost 3 feet above mean high water. The highest tide recorded at Fort Hamilton since 1893 was 6.1 feet above mean sea level. This occurred on April 11, 1918. The lowest tide observed at Fort Hamilton since 1893 occurred on February 2, 1908, when low water fell 6.5 feet below main sea level, or 4.1 feet below mean low water. The extreme range, therefore, as between the highest and lowest

tides observed at Fort Hamilton is 12.6 feet. Compared with the average range of 4.7 feet this extreme range is 170 per cent greater.

The characteristics of the tide at Fort Hamilton prove it to be very nearly the tide of the open coast. Thus, as compared with the tide at Atlantic City, we find the tide at Fort Hamilton to be some 15 per cent greater in range, with high water occurring 27 minutes later and low water 38 minutes later. At Atlantic City, too, the seasonal changes in the height of the tide are similar to those found at Fort Hamilton, sea level being lowest about the latter part of January and highest in August, the difference being five-tenths of a foot. A notable difference, however, between the tide at the two places is found in the relative periods of rise and fall. At Atlantic City the two periods are equal, both rise and fall occupying a period of 6 hours 12 minutes; but at Fort Hamilton, the period of fall is 21 minutes greater than the rise, this increase in period of fall giving evidence of the existence of features characteristic of river tides.

In the rising and falling of the tide in New York Harbor a very considerable mass of water passes through the Narrows—from the sea into the harbor on the flood and from the harbor into the sea on the ebb. This movement of the water in and out through the Narrows gives rise to tidal currents. The characteristics of these currents are shown in Figure 53, in which the upper curves represent in graphic form the velocities of the current expressed in knots or geographic miles per hour, as observed in the center of the channel off Fort Wadsworth on August 17, 1922. The velocities of the current are shown for three depths, 17 feet, 44 feet, and 70 feet, these depths being respectively two-tenths, five-tenths, and eight-tenths from the surface. The curves above the zero line of velocities represent the northerly or flood current, and the curves

below the zero line represent the southerly or ebb current. For comparison there is also shown the curve of the tide at Fort Hamilton on the same date.

It is to be noted that the current curves reproduce the general features of the tide curve. For the Narrows a comparison of the simultaneous tide and current curves shows that the slack of the current comes about midway between the times of high and low water; maximum flood current, or "strength of flood," occurs near the time of

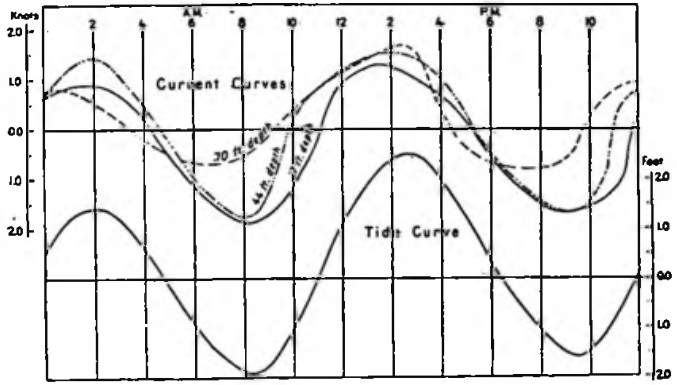


FIG. 53.—TIDE AND CURRENT CURVES, THE NARROWS, NEW YORK HARBOR, AUGUST 17, 1922

high water; and strength of ebb occurs about the time of low water. These time relations between current and tide are characteristic of the progressive-wave type of tidal movement, and we may therefore conclude that the tide in the Narrows is of the progressive-wave type.

An inspection of the current curves in Figure 53 brings to light the fact that the whole mass of water, from top to bottom, takes part in the tidal movement in and out through the Narrows. In two respects, however, the current curves for the different depths show unmistakable

differences: the velocity of the current at the three depths is not the same, and the turn from flood to ebb (and vice versa) takes place at different times. These differences are due primarily to the fresh water discharged into the sea through the Narrows.

The streams tributary to New York Harbor bring to it at all times a very considerable amount of fresh water. During the months of March and April, when freshet conditions prevail, somewhat more than 50,000 cubic feet of this fresh or nontidal water is brought down every second to the Narrows to be discharged into the sea. And even during the month of August when the fresh-water run-off generally reaches its minimum, the fresh-water discharge through the Narrows is more than 13,000 cubic feet per second.

Obviously, the effect of this fresh water running continually toward the sea is to increase the duration and velocity of the ebb or outgoing current. The 17-foot curve in Figure 53 brings this out clearly. But it is to be noted that the density of this fresh water is less than that of the saline water coming into the harbor from the sea. As a result the fresh water tends to keep toward the surface so that the effect mentioned should decrease from the surface downward. The 44-foot and 70-foot curves show this to be the case, the latter, indeed, indicating the greater duration and velocity for the flood current.

Since the current does not turn flood or ebb at the same instant from the surface to the bottom, it is evident that there will be times when the current near the bottom will be flowing in a direction opposite to the current near the surface. Thus from Figure 53 it is seen that at 10:00 A.M. on August 17, 1922, the current at the 17-foot depth was setting ebb, or southerly, with a velocity of about 1¼ knots, while at the same time the current at the 70-foot depth was setting flood, or northerly, with a velocity of

one-half a knot. It is to be observed, however, that at the time of strength the current from top to bottom is flowing in the same direction.

From day to day the tidal current undergoes changes which are related to the changes exhibited by the rise and fall of the tide. Full moon and new moon bring not only greater tides than usual but also stronger currents; likewise perigean tides mean stronger currents, while neap tides and apogean tides are accompanied by relatively weaker currents. In fact, from theoretical considerations the tidal current in the Narrows should vary in velocity as the tide varies in range, and this is borne out by the observations.

One other feature of the tidal movement in the Narrows should be mentioned, and that is the relation of the rising and falling tides to the flood and ebb currents. Figure 53 shows that from 2:00 A.M. to 8:20 A.M. the tide in the Narrows was falling; part of this time, however, the current was flooding—from 2:00 A.M. to 4:00 A.M.—and the rest of the time the current was ebbing. Similarly, from low water at 8:20 A.M. to high water at 2:30 P.M. the tide was rising; but during this period the current was ebbing part of the time and flooding the remainder of the time. And this state of affairs is characteristic of the tidal movement in the Narrows. On the first half of the rise, the current is ebbing, on the second half, it is flooding; on the first half of the fall, the current is still flooding, but during the second half of the fall, the current has turned ebb. The confusion that arises from identifying the rising tide with the flood current and the falling tide with the ebb current is clearly brought out by Figure 53.

The tide coming in through the Narrows progresses into Upper Bay at a rate dependent on the depth of the waterway, the rate of advance being given approximately by the formula $r=\sqrt{gh}$ in which r is the rate of advance, g the

acceleration of gravity, and h the depth of water. At St. George, near the entrance to Kill van Kull, the tide is 20 minutes later than at Fort Hamilton; at Governor's Island, at the mouth of East River, the tide is 25 minutes later; and in the entrance to Hudson River, abreast of Battery Place, it is 30 minutes later. Throughout Upper Bay the range of the tide is about 4.4 feet, or three-tenths of a foot less than in the Narrows. This decrease is not surprising in view of the considerable increase in the width of the tidal waterway north of the Narrows.

In its relation to the varying positions of sun and moon, and in its characteristics, generally, the tide in Upper Bay is very much like the tide in the Narrows. The currents likewise reproduce the features of the current in the Narrows and, like the tides, are later in Upper Bay. The velocities of the current, however, are generally less than in the Narrows, corresponding to the decrease in the range of the tide.

From Upper Bay the tide swings into the Hudson, the tidal and current phenomena of which have been discussed in previous chapters. Here it will be sufficient to summarize these phenomena in the 12-mile stretch to Spuyten Duyvil where the Harlem enters the Hudson. In this stretch the tide becomes later by one hour and decreases in range from 4.4 feet to 3.8 feet. Apart from these differences in time and range, the tide in the lower Hudson exhibits the features that characterize the tide at Fort Hamilton.

The current phenomena in the lower Hudson, like the tidal phenomena, are similar to those in the Narrows. In fact, from the Narrows, through Upper Bay, and the lower Hudson, the tidal and current phenomena are much the same. Throughout this stretch the tidal movement is of the progressive-wave type; and this conditions the characteristics of both tides and currents, except as they are

modified by the large amount of fresh water discharged through the Hudson.

Turning our attention now to the ocean tide that enters New York Harbor from the east, we note that this tide must traverse the waters of Long Island Sound before passing the eastern gateway to the harbor at Throgs Neck and entering East River. At Throgs Neck, the tide occurs 3¾ hours later than at Fort Hamilton and has a mean

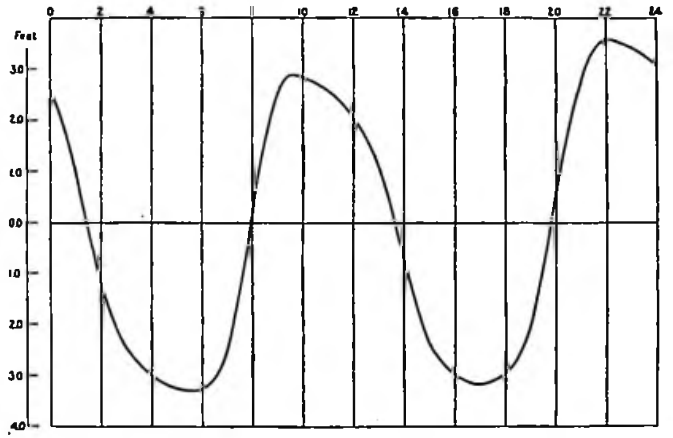


FIG. 54.—TIDE CURVE FOR THROGS NECK, THE EASTERN ENTRANCE TO EAST RIVER

range of 7.2 feet. The tide curve representing the average rise and fall is shown in Figure 54, with reference to the time of the moon's passage over the meridian of Fort Hamilton. Throughout this chapter the time of tide, when referred to the apparent revolution of the moon, will be taken with reference to the meridian of Fort Hamilton, since this procedure will make the time of tide at all places in New York Harbor directly comparable.

Comparing Figure 54 with the upper curve of Figure

51, which represents the mean curve of the tide at Fort Hamilton, it is seen at once that, apart from the difference in time and range, the tide curves for the two places exhibit differences in the character of the rise and fall. At Fort Hamilton the curve of the tide from high water to low water, and likewise from low water to high water, approximates closely to the cosine curve. At Throgs Neck, however, the rise and fall deviates from the cosine curve in that the tide curve is more rounded near the times of high and low water, giving a longer "stand" to the tide at those times. As regards changes in the rise and fall in response to the changing positions of the moon relative to earth and sun, the tide at Throgs Neck behaves much like that at Fort Hamilton. Spring and neap tides and perigean and apogean tides show percentage increases and decreases similar to those at Fort Hamilton.

The currents in the channel abreast of Throgs Neck are much weaker than the currents off Fort Hamilton. Instead of velocities up to 2 knots, the currents here have velocities which, at strength, are barely over 1 knot. The characteristics of the tidal currents in the eastern gateway and the time relations of these currents to the tide are shown in Figure 55, which is a plotting of the observations made in the center of the channel between Throgs Neck and Willets Point on August 6, 1922. The current velocities at three depths are shown—at 22 feet, 55 feet, and 88 feet. These depths represent, respectively, distances from the surface two-tenths, five-tenths, and eight-tenths of the depth at that point. The curves above the zero line of velocities represent the westerly flowing current, and the curves below the zero line represent the easterly flowing current. For comparison, the curve of the tide at Throgs Neck on the same day is plotted in the lower part of the diagram.

Exactly the same time and velocity scales are used in

Figure 55 as in Figure 53. A glance at the two figures shows that, despite the fact of a considerably greater range of the tide in the eastern gateway, the currents here have a velocity approximately half that at Fort Hamilton. Attention is called to this fact to emphasize the necessity of distinguishing between tide and current at any place. To say that at a given point there is a "strong tide" is not

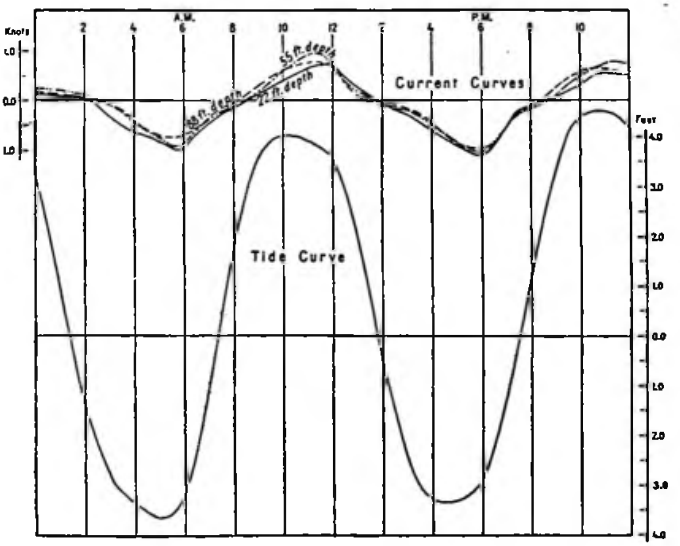


FIG. 55.—TIDE AND CURRENT CURVES, EAST RIVER, NEW YORK, OFF THROGS NECK, AUGUST 6, 1922

very enlightening, for what is probably meant by such a statement is not that the tide has a relatively large range but that the current has considerable velocity.

We have now followed the ocean tide to the western entrance of the East River in Upper Bay, and to the eastern entrance at Throgs Neck. The tides in the Upper Bay differ considerably from those at Throgs Neck in

time, range, and characteristics of rise and fall. To what phenomena will the differences in the tides at the two ends of East River give rise? For, surely, it is but reasonable to expect that the resulting tidal phenomena in the East River will differ very considerably from those in the Hudson.

From Upper Bay, East River carries the tide northeastward, while the Hudson River carries it northward. The average depth of East River is approximately the same as that of the lower Hudson from the mouth to Spuyten Duyvil, being in both cases about 30 feet. The distances, too, are very nearly the same, the channel of East River from the Battery to Throgs Neck measuring 14 geographic miles, while that of the Hudson River up to Spuyten Duyvil measures 12 miles. The tide, however, takes but an hour to traverse the 12 miles up the Hudson, while in the 14 miles of East River there is a change of 3¼ hours in the time of tide.

It is to be noted, too, that in the Hudson the time of the tide changes in a uniform manner from the Battery to Spuyten Duyvil, each mile being traversed in approximately 5 minutes. In East River this is not the case. In the first 6 miles—from the Battery to the northern end of Blackwell's Island—the tide becomes later by 2 hours, the change taking place in a relatively uniform manner; that is, for each mile there is a change of about 20 minutes in time of tide. From the end of Blackwells Island through Hell Gate the time of tide changes very rapidly, this stretch of a mile and a half showing a change of 1½ hours, or a change at the rate of 60 minutes for each mile. For the next 3 miles there is practically no change in the time of tide, and for the remaining 3½ miles to Throgs Neck there is a change in the opposite direction, the tide actually becoming earlier by a quarter of an hour.

Not only in time but also in range of tide do the two

waterways exhibit marked differences. From a range of 4.4 feet at the Battery the tide in the Hudson decreases to 3.8 feet at Spuyten Duyvil, this decrease of 13½ per cent in the range taking place in a uniform manner throughout

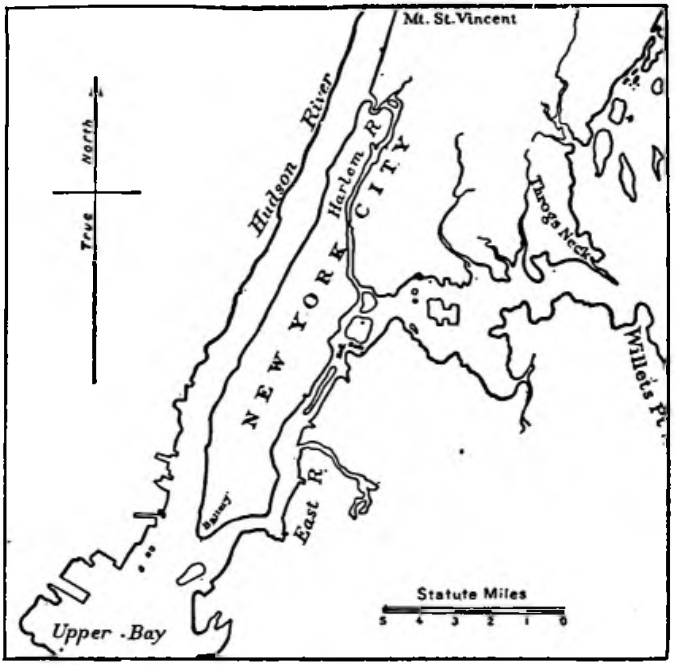


FIG. 56.—EAST RIVER AND HUDSON RIVER, NEW YORK HARBOR

the stretch of 12 miles, or at the rate of a little over 1 per cent per mile. In East River, from the range of 4.4 feet at the Battery the tide increases to a range of 7.2 feet at Throgs Neck—an increase of 63½ per cent—but this increase is not uniform over the stretch of 14 miles.

For the first 3 miles from the Battery, the tide, instead of increasing, actually decreases almost one-half of a foot,

the rise and fall of the tide a little north of the Navy Yard being exactly 4 feet. After this initial decrease the tide begins to increase in range, mounting steadily each mile until at Throgs Neck 7.2 feet is attained. In the first 3 miles, therefore, the tide in East River decreases at the rate of 3 per cent per mile, while for the remaining 11 miles there is an increase in the range at the rate of 7¼ per cent per mile.

Now it is important to note that this behavior of the tide in East River, so strikingly different in regard to both time and range from the tide in the waters around it, cannot be ascribed to any peculiar hydrographic or geographic features of the waterway itself. As we shall see later in the discussion of the character of the tidal movement, the time and range of the tide in East River are conditioned by the fact that East River is a short strait connecting two bodies of water having tides independent of each other.

As regards the characteristics of the rise and fall of the tide throughout East River, it may be said that from the Battery to Hell Gate the tidal curves are very much like those for Fort Hamilton shown in Figure 51, while in the stretch from Hell Gate to Throgs Neck the tide curves exhibit the more rounded appearance of the tide at Throgs Neck shown in Figure 54.

If we now examine the behavior of the currents, we find that in this respect, too, the Hudson and East Rivers differ. In the Hudson the current, like the tide, becomes later in going northward from Upper Bay; but in the East River conditions are reversed, the slack and also the strength of the current occurring earlier as we go upstream from Upper Bay.

It is to be recalled that the time of *tide* becomes later from Upper Bay to Throgs Neck by 3¼ hours. Since the time of *current* in the same stretch becomes earlier, it is obvious that the relation of the time of current to

the time of tide varies considerably from one end of the river to the other. At its western end, off the Battery, the current that sets into East River attains its strength about 4½ hours after local low water. Abreast of Throgs Neck, at the eastern end of the river, this same current attains its strength about half an hour after low water.

The swiftest currents found in New York Harbor occur in East River. The average velocity at time of strength throughout the stretch from Governor's Island to Throgs Neck is about 2½ knots, but this velocity varies from place to place in accordance with the variation in cross section of the waterway. In Hell Gate and in the channel to the west of Blackwells Island the area of the cross section is greatly reduced, and here velocities of 5 knots and more are of frequent occurrence.

Now it is obvious that the tidal movement in the East River differs from the progressive-wave movement that characterizes the tide in the Hudson. It is also different from the stationary-wave movement that is exhibited in certain bays and sounds. How can we arrive at an understanding of the tidal movement in this waterway?

In the discussion of tidal phenomena it is frequently of advantage to employ terms that relate to wave motion. Thus, we speak of the advance of the tidal wave, of high water as the crest of the tide wave, and of low water as the trough; the two principal types of tidal movement are designated as the progressive-wave and stationary-wave types. In terms of wave motion, therefore, the tidal movement in the East River is frequently described as resulting from the "interference of the two tidal waves, the one entering from Upper Bay and the other from Long Island Sound."

Notwithstanding the advantages accruing from the conception of the tide in terms of wave motion, its application to the tidal movement in the East River is unfortunate.

To ascribe the tidal and current phenomena in this waterway to the "interference of tide waves" relegates the whole matter to the mathematician and enshrouds the mechanism of the movement, which in reality is quite simple, with a terminology which makes it inexplicable in nonmathematical language. Moreover, it frequently leads to absurd statements; as, for example, that "the two tide waves, entering East River from Upper Bay and from Long Island Sound, meet in a high water crest somewhere east of Whitestone Point."

The movement of the water in East River is brought about by the fact that part of the time the level of the water in Upper Bay is higher than in Long Island Sound and part of the time it is lower. East River thus acts merely as a channel through which the water flows from the body having, temporarily, the higher level to the one having the lower level. In other words, the movement of the water in East River is primarily hydraulic in character.

From simple hydraulic considerations, it is evident that when the level of the water at the two ends of East River is the same no movement of the water can take place; but as soon as a difference in level arises, the water will flow towards the body having the lower level. Furthermore, the velocity of this current increases as the difference in level increases, the maximum velocity or strength of current coming at the time of greatest difference in level. Hence, if we plot the heights of the tide at the two ends of East River, the lines joining simultaneous heights will indicate by their slope the difference in level; and, if the reasoning above applies, we should expect slack water about the time of zero slope and strength of current about the time of maximum slope.

In Figure 57, the simultaneous heights of the level of the water at the two ends of East River are represented

for each hour of the tidal rise and fall, time being reckoned from the instant of the moon's passage over the meridian of Fort Hamilton. On the vertical line to the left the hourly height of the tide is plotted as taken from the mean tide curve for the Battery, while on the vertical line to the

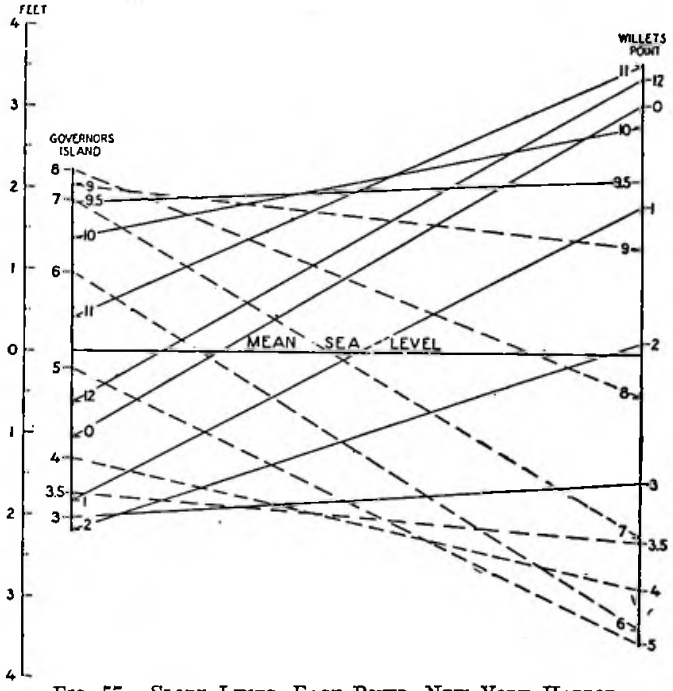


FIG. 57.—SLOPE LINES, EAST RIVER, NEW YORK HARBOR

right the hourly heights are taken from the mean tide curve for Throgs Neck. In both cases the heights are measured from mean sea level, indicated by the heavy horizontal line, the scale of heights in feet being shown to the left. The figures along the vertical lines denote the hours to which the indicated heights apply, the hours and

heights being placed to the left of each vertical line for the rising tide and to the right for the falling tide. The slope lines—the lines joining simultaneous heights—are drawn full when the slope is from Long Island Sound to Upper Bay and dashed when the slope is in the reverse direction.

If the movement of the water in East River is due to the difference of level at the two ends, as pictured in Figure 57, then high water at any given place in the river is the highest point in the slope lines passing that place, and low water is the lowest point in the slope lines. Furthermore, the times of high and low water at that place are given immediately by the respective slope lines on which the highest and lowest points are found. For example, about one-fourth the distance between the Battery and Throgs Neck, the highest point is on the 9-hour slope line, and the lowest point is on the 3-hour line. On the scale in feet to the left these points are respectively 2 feet above and 2 feet below mean sea level. Hence the height to which high water at this point rises is 2 feet above sea level, and the depth to which low water falls is 2 feet below sea level, giving a range at that place of 4 feet—less, be it noted, than the range at the Battery. The time of high water with respect to the moon's passage over the meridian of Fort Hamilton, is obviously 9:00 o'clock and the time of low water 3:00 o'clock.

From the diagram it is evident that the highest points of the slope lines define the curve of the high waters in the river, while the lowest points define the curve of the low waters. The diagram, therefore, shows at once that from the Battery the range of the tide decreases for about one-quarter the way to Throgs Neck, after which it increases. And this, it is to be recalled, we found to be the case. Furthermore, the diagram gives the smallest range in the river as 4 feet, which figure is exactly that found by actual

observation. This decrease of the range in the first 3 miles of East River—which, upon being shown by observation to exist, has generally been ascribed, in a vague way, however, to the "interference of tide waves"—is thus shown to follow immediately by considering the movement of the water to be hydraulic.

One other feature of the tides in East River merits more than passing mention because of its practical importance, and that is the line of low water in the river. This line, instead of being a straight line between the Battery and Throgs Neck, as might be expected at first thought, is shown by the hydraulic diagram of Figure 57 to be a curved line, convex toward the surface. This is characteristic of the low-water line in a waterway which connects two independent tidal bodies of water, and its practical importance lies in the fact that in the deepening of such a waterway a considerable saving in excavation may frequently be made. For, instead of dredging the bottom to a uniform depth below mean sea level, advantage may be taken of the fact that the low-water line for some distance lies above the straight line joining the low-water points at the two ends.

The current phenomena in East River likewise may be derived from the hydraulic diagram of Figure 57. Thus, the times of slack and of strength of the current should obviously be very nearly simultaneous throughout the river, the former occurring when the difference in level at the two ends is zero and the latter when the difference is greatest. From Figure 57 we find that, with reference to the time of the moon's passage over the meridian of Fort Hamilton, the water at both ends is at the same level a little after 3:00 o'clock and also at 9:00 o'clock. From observations it is found that the time of slack water changes but little throughout the river, the average time being a few minutes later than given by the diagram.

The greatest slope towards Upper Bay is shown by the 0-hour line, and hence the strength of the westerly current should come about 0:00 o'clock; the greatest slope towards Long Island Sound is shown by the 6-hour line and the strength of the easterly flowing current should come about 6:00 o'clock. Observations show that these times are approximately the times of strength of the respective currents throughout the river.

It is obvious that the simple hydraulic considerations on which Figure 57 is based can give only a first approximation to the tidal and current conditions existing in East River. No account has been taken of the varying depths in the waterway, of the differences in width and changes in direction, nor of the effect of the water coming through Harlem River. The slope lines are drawn as if the channel from the Battery to Throgs Neck ran straight for the stretch of 14 miles with unvarying width and depth. However, notwithstanding the modifications that must arise from the fact that East River deviates from the uniform channel assumed in Figure 57, the principal tidal and current phenomena in the river are easily derived by considering the movement of the water in East River as hydraulic. The outstanding advantage of regarding the phenomena from the hydraulic point of view is that it permits us to visualize the mechanism of the movement of the water.

It will not be necessary to discuss in detail the tidal and current phenomena in Harlem River and Kill van Kull. For each of these two waterways is, like East River, a short strait connecting two bodies of water in which the tides differ in time and in range. The tidal and current phenomena in these two waterways will therefore be similar to the phenomena found in the East River. It is to be noted, however, that the differences in time and range of tide at the ends of Kill van Kull and Harlem River are

considerably less than at the ends of East River, and therefore the tidal and current phenomena in these two waterways are not as striking as in the East River.

In consequence of the tide in New York Harbor, there is a ceaseless interchange of water between the harbor and the sea. For a period of approximately 6 hours, on the flood, the sea pours its waters in the harbor, and in the following period of 6 hours the harbor waters move toward the sea.

The current does not turn to flood or ebb at the same time throughout the various parts of the harbor. It turns earliest in Kill van Kull and latest in the Hudson, but this difference is only 3 hours. For the remaining 3 hours of the flood or ebb cycles the current will be flooding or ebbing throughout the whole harbor. Figure 58 is a schematic representation of the current in the component parts of New York Harbor one hour before the time of high water at Fort Hamilton. At this time the current throughout the harbor is running flood at very nearly its strength. Through the Narrows the ocean waters are now pouring into Upper Bay with the greatest force, carrying into the harbor at this time about three-quarters of a million cubic feet each second; in Kill van Kull the strength of the current occurred 1 hour before; in East River the current is just about at its strength setting toward Long Island Sound; in the Harlem, likewise, the current is about at its strength flowing into East River, while in the Hudson the current is gathering momentum for the strength which will occur about 1½ hours later.

Figure 59 represents the conditions 6 hours later, or one hour before the time of low water at Fort Hamilton. Throughout the harbor the current is running ebb at very nearly its strength. For 1½ hours the Hudson has been returning the tidal water received on the previous flood together with the fresh-water run-off from the thousands

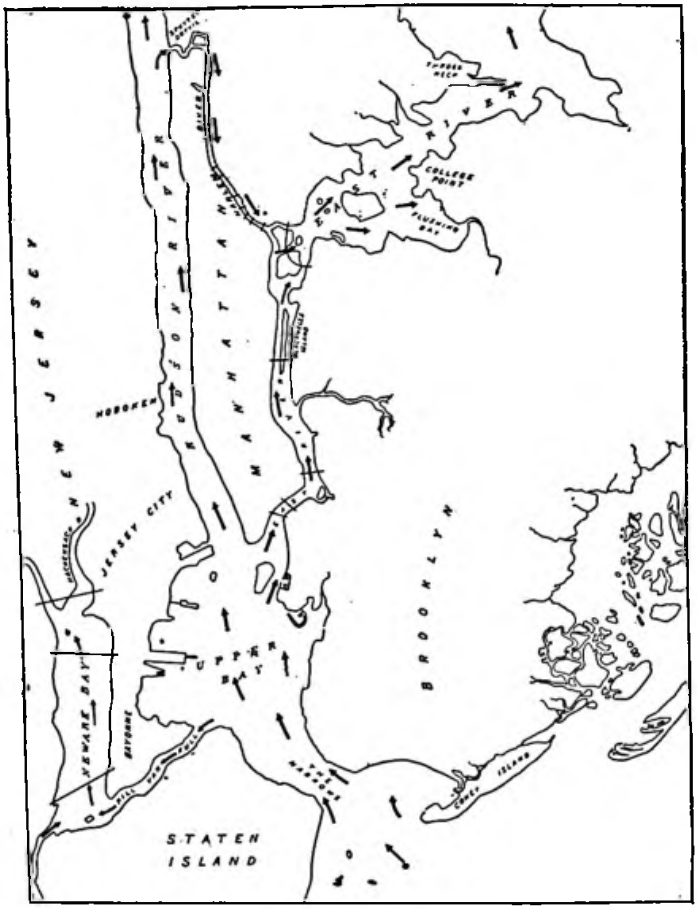


FIG. 58.—FLOOD IN NEW YORK HARBOR

of miles of territory it drains; but the strength of the ebb in the Hudson is still 1½ hours away. The Harlem is now running at its strength carrying the waters of the East River toward the Hudson, while in the former the current is

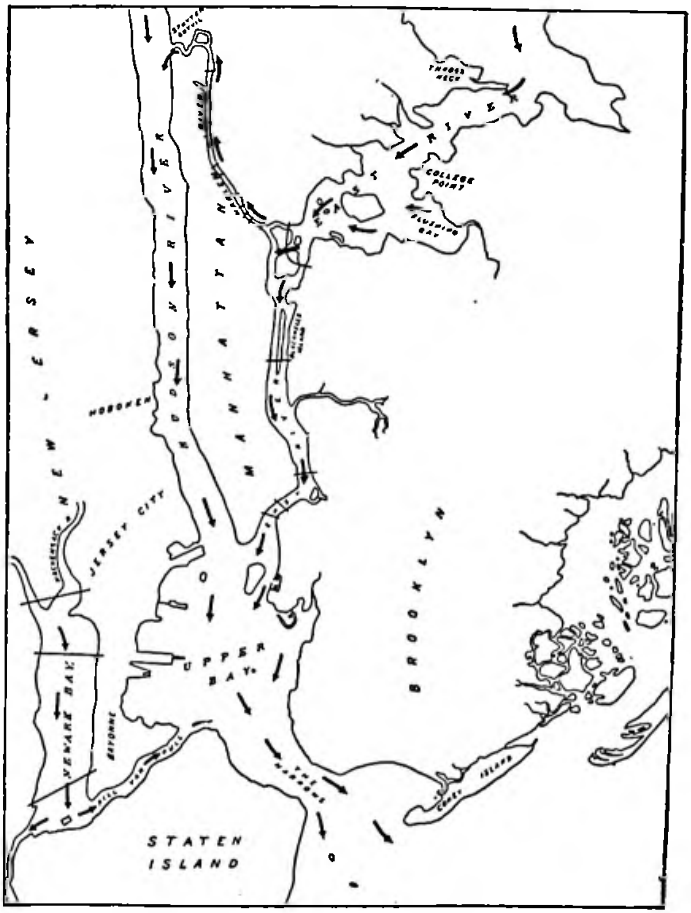


FIG. 59.—EBB IN NEW YORK HARBOR

setting toward Upper Bay, the strength of the current having occurred about half an hour previously. In Kill van Kull the strength of the ebb is 1½ hours past, but the waters of Newark Bay are still flowing into Upper Bay.

Gathering the ebb waters from the Hudson River, East River, and Kill van Kull, Upper Bay pours them out to sea through the funnel of the Narrows, though it is to be noted that at this time—1 hour before low water at Fort Hamilton—the current through the Narrows is still about an hour from its strength.

For the port of New York and for the communities situated along the banks of the tidal streams tributary to the harbor this interchange of water consequent on the tide is of importance in connection with the maintenance of the channels and the disposal of sewage. In the maintenance of the channel of a waterway, tidal currents are of importance since sand will be moved along the bottom of a channel by a current of half a knot, while stones about 1½ inches in diameter will be carried along by a current of 3½ knots. In the disposal of sewage the tidal action is of a twofold character. In the first place, tidal currents carry some of the solid and liquid ingredients out to sea; and, secondly, the uncontaminated sea water brought into the harbor on the flood aids in assimilating the sewage and thus converting it into inert mineral matter.

The observed velocities of the current furnish us the necessary data for computing the volume of water passing in and out the harbor on flood and ebb. These computations bring out the fact that during the tidal cycle of 12½ hours, on the average, 11 billion cubic feet pass into the harbor through the Narrows on the flood, and 12¼ billion cubic feet pass out on the ebb. In a day, therefore, there is a net seaward discharge through the Narrows of about 2½ billion cubic feet. Obviously this excess of the ebb volume over the flood is due primarily to the drainage waters that find their way to the sea through the streams that are tributary to New York Harbor.

From the Upper Bay the tide carries 5¾ billion cubic feet into the Hudson during the flood, while during the

ebb the Hudson discharges into the bay 6¾ billion cubic feet, the fresh-water run-off being a little in excess of 1 billion cubic feet during a tidal cycle of 12½ hours, or 2 billion cubic feet per day. Through Kill van Kull the flood carries from New York Harbor into Newark Bay 1½ billion cubic feet, and the ebb returns into the harbor very nearly 100 million cubic feet more; so that in a day Newark Bay pays a tribute of almost 200 million cubic feet of water to New York Harbor. Through the Harlem, Hudson River receives on each ebb about 30 million cubic feet more than it returns on the flood, and East River likewise pours into Upper Bay on the ebb about 100 million cubic feet more than it returns on the flood, so that on the interchange between Long Island Sound and Upper Bay, Long Island Sound, in a day, pours into New York Harbor nearly 300 million cubic feet of water more than it receives.

From the large quantity of water circulating in the various channels of New York Harbor, an exaggerated notion of the flushing action of the waters in the harbor is frequently entertained. It is to be borne in mind that for the greater portion of the harbor the water that is returned on the flood is not clean sea water, but harbor water which had been carried to a lower portion of the harbor on the preceding ebb. It is to be recalled that with a current which at strength has a velocity of 2 knots a particle will be carried a distance of about 8 miles during the period of a single flood or ebb. If, therefore, the flood and ebb velocities were equal, a particle would merely move a given distance downstream on the ebb and return the same distance upstream on the flood. It is the excess of the ebb over the flood that finally carries a particle from the upper reaches of the harbor out past the Narrows and into the sea.

In the circulation of the waters in the harbor it is there-

fore the excess of the ebb volume over the flood that is the important feature. Two causes contribute to this ebb excess. The first and principal cause is the fresh-water run-off from nearly 15,000 square miles of territory that drains into the Atlantic through New York Harbor, and the second is the excess of the flow coming from Long Island Sound which circulates through the Harlem, Hudson, and East Rivers and thence through the Narrows out to sea.

CHAPTER XVIII

SOME EFFECTS OF THE TIDE

THE great ports of the world, almost without exception, are situated on tidal waters. New York, Philadelphia, San Francisco, London, Liverpool, Hamburg, Antwerp, Bordeaux, and many others may be cited. Some of these ports are many miles from the sea—London is 50 miles from the mouth of the Thames, Hamburg is 75 miles up the Elbe—and to reach them the ship-borne traffic of foreign and domestic commerce must traverse the tidal waters that connect these ports with the open ocean. These tidal waters thus serve as highways to the seven seas, and for each port the tidal régime in the channel that connects it with the open ocean is a matter of considerable importance.

In the days of the shallow-draft ship the highway leading from the sea did not possess the importance attaching to it to-day. Cologne, 210 miles up the Rhine, was in medieval times one of the great Hanseatic seaports, although the channel at that time was less than 6 feet in depth. But the introduction of steam as the propelling power and the advent of the iron and steel ship, together with the enormous increase in world-wide trade brought about by the industrial development of the past century, make the channel leading to the sea a feature of first importance in the modern harbor.

The time required for a sailing vessel to cross the ocean, together with its dependence on wind and weather, made impossible the maintenance of an exact schedule for arrival and departure. If the channel leading to the harbor was

not sufficiently deep to permit entrance at low water the delay of 6 hours awaiting high water was not a serious matter, for this delay represented but a very small fraction of the time of the voyage. Now with the time of crossing the ocean counted in days and with a transatlantic liner representing an initial investment of several million dollars, a delay of as much as a quarter of a day at each end may mean the difference between profit and loss. For the profit from the modern leviathan—costly to build and costly in operation as a unit—depends upon the number of trips it can make a year.

The advent of the iron and steel steamer removed the limitation on the size of ships that wood construction had imposed. The larger ship is also the more profitable one to operate, provided the traffic is sufficiently large. And where the stream of traffic was sufficient, as in the transatlantic trade, the shipping industry responded with larger and larger vessels. The *Leviathan,* one of the largest ocean liners, may be taken as type. With a length of nearly 1,000 feet, a width of 100 feet, and with a displacement of over 66,000 tons, she carries a crew numbering 1,100, and requires a channel depth of about 40 feet. As she steams for the harbor of New York at her normal speed of 600 miles per day, the *Leviathan* is a far cry from the *Half Moon* of 80 tons' burden and draft of 7 feet, which with a crew of 18 or 20 men Henry Hudson sailed into the same harbor but a little over three centuries ago.

The tide is the bond that keeps the channel leading to the port in intimate contact with the open sea and its vivifying influences. And the extent of rise and fall of tide in the channel is the factor that frequently determines the type of harbor that must be developed. If the rise and fall of the tide, or more precisely the range of the tide, is over 15 feet the closed-dock type of harbor generally results. This is exemplified by London with a tidal

range of 16 to 21 feet and by Liverpool with a range of 15 to 27 feet. A large vessel rising and falling with the tide 15 feet or more twice each day presents a number of problems in loading and unloading. Furthermore, the erection of suitable wharf structures would entail considerable expense. The result in such cases is therefore the closed-dock harbor in which the docks are entered through locks open for a few hours about the time of high water.

Where the rise and fall of the tide is less than 15 feet and the channel is sufficiently deep, the open-basin type of harbor results. In this case piers may be constructed directly into the channel, and a vessel may dock at all stages of the tide, obviating the delay arising in the closed-dock harbor which can be entered or left only near the time of high water. In this regard all the great American ports are fortunately situated, the range of the tide generally being less than 10 feet.

To permit the deep-draft vessels of modern commerce to enter port enormous sums have been expended in the effort to maintain a sufficient depth in the channels connecting the seaports of the world with the open ocean. The ideal channel is one sufficiently deep to permit all vessels to enter port at any stage of the tide. Where this ideal was impracticable of realization, it has frequently been found necessary to construct a subport or twin port closer to sea somewhere along the channel. Obviously this entails extra expense in bringing up the ship-borne traffic, either by rail or lighter, from the lower subport. Despite this, examples of twin ports are not uncommon in Europe. To mention but the more prominent ones we may cite London and Tillbury, Hamburg and Cuxhaven, Bremen and Bremerhaven.

The tide gives ports located on tidal rivers a number of advantages over those located on nontidal rivers. In

capacity for improvement nontidal streams rate low, for in such streams the quantity of water available is generally limited and is, moreover, subject to fluctuation throughout the year, since it depends very largely upon the amount of rainfall or snowfall. In tidal streams, on the other hand, the amount of water that may be obtained is practically unlimited, for, in consequence of the tides, there is direct and intimate connection with the boundless waters of the open sea.

Numerous instances of the great improvements made in tidal streams can be mentioned. As an example we may cite the case of the river Clyde on the west coast of Scotland. As rivers go, the Clyde is a comparatively small stream and in its original state was fordable in summer 12 miles below Glasgow. Being a tidal river, however, the Clyde has responded to improvements and now it is the greatest shipbuilding center in the world and allows the largest vessels to steam up to Glasgow.

In shallow streams the rise of the tide gives added depth to the channel and thus makes accessible to shipping a greater stretch of territory. Certain harbors are, at their entrances, obstructed by bars with limited depths of water over them. Such harbors would be accessible only to shallow-draft vessels were it not for the tide which at high water gives added depth and makes possible the crossing of the bar by vessels of greater draft. On the other hand the tide makes unfit for agricultural purposes large areas of low-lying land along the coast, flooding such lands at high water with the briny waters of the open sea.

Water containing salt in solution has a lower freezing point than fresh water. Hence, by bringing in on the flood the salty water from the ocean, the tide lowers the freezing point of harbors situated on tidal streams and thus keeps these harbors open in the winter even though

the upper nontidal reaches of these streams become icebound. Thus, New York Harbor on the lower Hudson is open all winter, while the upper Hudson is icebound for several months. The importance of this lowering of the freezing point is apparent from the fact that the harbor of St. John's on the coast of Newfoundland is open during the winter while the upper reach of the Hudson River, in much lower latitudes, is frozen over.

It is due to tidal influence, too, that the material brought down by continental streams is gradually dispersed into the open sea. Were it not for the action of the ebb current which increases the velocity of the river current seaward, this material would deposit in the inshore waters and gradually shoal them. As it is, the river current, which decreases considerably as it approaches the sea, is reinforced by the tidal current on its ebb and thus sweeps the material out to sea, where it is dispersed by the waves.

Rivers which empty into a body of water swept by the tide are generally free from bars. This is due to the fact that the ebb current sweeping past the mouth of the river carries out any sediment that may have been deposited there by the river waters. For example, the rivers leading into Chesapeake Bay have no bars, for the tidal current moving in and out the bay, while in itself not very strong, is sufficient to prevent the accumulation of material at the river mouth; and in coöperation with the seaward flow of the fresh water the ebb current sweeps this material out to sea. In this connection it is to be remembered that a current with a velocity of half a knot will drive sand along the bottom; with a velocity of 1 knot it will move fine gravel, while with a velocity of 3½ knots it will move stones about 1½ inches in diameter. The tides thus are of considerable importance in the maintenance of the channel that leads to the sea.

The disposal of the sewage of a large industrial center

is a matter of importance and one that involves the expenditure of large sums of money. Generally the sewage is conducted into some neighboring body of water. Now, if this water is in motion, the sewage is carried away and so greatly diluted as to have no harmful consequences. As the saying goes, "moving water purifies itself." But if the sewage is conducted into a body of still water, the water becomes polluted and may give rise to serious consequences. In this regard therefore the tides have a beneficial effect. Indeed, as was pointed out in connection with flood and ebb in New York Harbor, in the disposal of sewage the tidal action is of a twofold character. The tidal currents not only carry some of the liquid and solid ingredients of the sewage out to sea, but they also bring into the harbor the uncontaminated waters from the open ocean, which aids in assimilating the sewage and thus converts it into inert mineral matter.

On the open coast the tide becomes an ally of the disintegrative marine agencies. The coast line, as the term is ordinarily used, is supposed to mark the boundary between land and sea, and on the small-scale maps with which we are familiar it has a deceptive appearance of precise location. In reality, however, the term is a misnomer for land and sea meet, not in a line, but in a zone marked by shifting boundaries of varying width. This zone belongs now to the sea and now to the land. At low water the land extends farther seaward and at high water the sea encroaches on the land. At times of neap tides the width of this zone is much reduced, while at times of spring tides it has its greatest width. This zone is frequently called the foreshore.

It is the tide that gives the coast line an added dimension and converts it into a zone. And in doing this the tide becomes an agent of geologic change, for the foreshore is the scene of constant warfare and change. Here wave

and current have their greatest effect in their attack on the land. At one point, under the constant beat of the waves, the land gives way slowly to the sea; at another point the turbid waters are relieved of their load of sediment to form a bar or spit which the land finally redeems from the sea.

In the shore changes along the coast it must be recognized that the tide is not the primary agent, for the actual wearing away of the land is the work of the waves, and more especially of storm waves. The rôle played by the tide is that of vehicle for the waves, providing for them a wider field of action. This fact requires emphasis, for not infrequently one meets with the assumption that shore changes along the coast are in some way due primarily to tidal action. It is to be remembered that, except in the restricted entrances to coastal bodies of water, the tidal current near the coast is very weak. On the open coast the tide as an agency of change plays the secondary rôle, but in inland bodies of tidal water it frequently assumes the principal rôle. For such bodies of water are generally so restricted in width as to make it impossible for storm waves to beat against their shores. But it is in these bodies of water that tidal currents attain velocities sufficient to move sand and even matter consisting of larger particles.

Another effect of the tide—one which may be of great importance in the future—is the energy inherent in its rise and fall. This has been discussed in detail in the chapter on Tidal Energy and so need only be mentioned here. On the biological side, too, it is obvious that the effects of the tide are of significance. For to the plant and animal life in tidal waters the distribution of temperature and salinity is of the highest importance. And in this distribution, especially in tidal waters along the coast, the tidal movement plays an important part.

It will be noted that the effects of the tide discussed in this chapter are those of a local character. That is, a particular effect may be of importance at one place but of little or no importance at another, depending on features of a local nature. There are, however, certain consequences of the tides that embrace the whole earth and it is these effects that we shall consider in the next chapter.

CHAPTER XIX

TIDE AND TIME

In our consideration of the various phenomena of the tides in the previous chapters, there was one feature which, if mentioned at all, received but scant attention, and that is the tidal friction involved in the movement of the tide. This matter, however, merits very considerable attention, for tidal friction opens a vista that permits us a glimpse of the very remote past in one direction, and of the far-distant future in the other. In fact, a consideration of tidal friction will prove that tide and time have a more intimate connection than is expressed in the old adage that "time and tide wait for no man."

Wherever motion occurs on our earth, we know that it must be accompanied by friction; and to this general rule the tides are no exception. In the motion of the waters brought about by the tides, friction is produced by the movements of the particles of water over each other, by the movement of the water over the beds of the oceans, seas, and rivers, and by the movement of the water on the shelving shores along the coast. The mathematician, however, has recently shown that the chief offenders in this matter are the shallow seas like Bering Sea, the Yellow Sea, and the Irish Sea.

Obviously, tidal friction consumes energy, and the question arises as to the store of energy that is drawn upon. Now, although it is the moon and sun that give rise to the tides, it can be shown that the energy consumed by the tides is drawn not from the moon or sun but from the

earth, and that this energy comes from the energy the earth possesses in virtue of its rotation. Tidal friction thus acts as a sort of brake on the rotating earth, compelling it to move more slowly, and hence increasing its period of rotation. In other words, through friction the tides are increasing the length of the day.

That tidal friction should draw on the energy of the earth is not surprising, for even from general considerations we can arrive at the same conclusion. The rotating earth may be regarded as a moving body subject to the friction of the tides. The effect of friction is to retard movement and tidal friction must therefore retard the rotation of the earth and thus increase the length of the day.

The stock of energy possessed by the earth is so enormously great as compared with the friction produced by the tide that it is only by a very small quantity that the day is lengthened, even over a long period of years. And while there appears, as yet, no way of accurately measuring the lengthening of the day brought about by tidal friction, we can estimate from the records of ancient eclipses that this must now be something like the thousandth part of a second in a century, or at the rate of about one minute in six million years. It is therefore altogether too minute to be detected by any means at our command. There is, however, no question that the friction of the tides tends to produce this lengthening of the day, and the interest of it lies not in its effects within recent times, but rather in the glimpses that it affords us of the earth in the remote past and in the distant future.

For no matter how minute the *rate* of lengthening of the day may now be, it is to be kept in mind that tidal friction has been operating for untold ages, so that the cumulative effects must by this time be considerable. Furthermore, as we shall see, during the earlier history of our earth the frictional effects were far greater than

they are now. With tidal friction as a starting point, the brilliant English mathematical astronomer, Sir George Darwin—son of Charles Darwin, the great naturalist—has built up an exceedingly interesting and very plausible theory as to the early history of the earth and moon.

Before taking up this theory, we must consider for a moment what effect the tides have on the moon. For, in accordance with the principle of action and reaction being equal and in opposite directions, we should expect the tides also to influence the moon. And as a matter of fact this is the case; the reaction of the earth's tidal friction on the moon is to make the moon recede from the earth and at the same time to lengthen the period of the lunar month.

The reasoning from which these conclusions are derived is of a mathematical character and is altogether too involved to reproduce here. But it is similar mathematical reasoning which the astronomer makes use of in the calculation of the movements of the heavenly bodies; and there is no question as to the validity of these conclusions, that as a result of tidal friction the day and the month are becoming longer and the moon is receding from the earth. With regard to the past history of the earth we are therefore led to conclude that the length of the day was less, increasing to its present value from some smaller value. Furthermore, in the past the moon must have been nearer the earth, its present distance representing an increase from some lesser distance.

With these conclusions as to the effects of tidal friction, suppose we go backward in time and attempt to trace the early history of the earth and moon. Let us go so far back that the whole life of man on this earth is but a day in the reckoning—back to the time when according to the nebular hypothesis our earth was a molten mass. Obviously as we travel backward in time, we are undoing the effects

of tidal friction, the day is becoming shorter and the moon is approaching nearer the earth.

But it is to be noted that when the moon was nearer the earth, the tides raised were much greater than now and not merely in simple proportion to the distance from the earth. For, as we have found, the law of the tides is such that the tide-producing power of a heavenly body varies as the cube of its distance from the earth. Hence with the moon at half her present distance from the earth, the power to produce tides would have been 8 times as great. The effects of tidal friction would, therefore, on account of the increased tides, have been multiplied in the same ratio. But because of the moon itself then being nearer, another factor is introduced, which further multiplies the frictional effects fourfold. In other words, the efficiency of tidal friction in increasing the length of day and month and the distance between earth and moon varies inversely as the sixth power of the distance.

These conclusions, it should be emphasized, are not speculations, but are based on well-known physical laws. And as a result, when the moon was half her present distance from the earth the effects of tidal friction were $2^6=2\times2\times2\times2\times2\times2=64$ times as great as they are now. And when the moon was one-tenth her present distance from the earth, the effects of tidal friction were $10^6=1{,}000{,}000$ times as great as now.

We find, therefore, that although the effects of tidal friction now are excessively small they were enormously greater in the remote past when the moon was nearer the earth. It follows, too, that in the remote past the moon was close to the earth, for the time demanded by tidal friction to bring the moon to its present position from a position close to the earth is not infinitely long. In fact, it has been calculated, in accordance with the above considerations, that although this period must be measured in

millions of years, it is no greater than the age assigned to the earth by the geologist and physicist who base their estimates on totally different data.

In tracing backward in time the history of earth and moon by means of mathematical formulas, the astronomer arrives at the conclusion that millions of years ago, when the moon was very close to the earth, almost touching its surface, the day had a length of 4 hours; that is, the earth rotated about its axis in a period something like 4 hours. If now the question is asked, "What was the condition of affairs prior to that time; how came the moon and earth to be so near each other?" the mathematician frankly says that calculation does not help. In attempting to unravel what happened before that time, he is compelled to resort to speculation; but this speculation he bases on the known physical laws that govern the universe.

Frequently the newspapers carry the story of a farmer who, thinking to save time and labor, belted a grindstone directly to his automobile—with the consequence that the grindstone flew to pieces, one of these striking the farmer and killing him. He had not taken into consideration the fact that a rapidly rotating body gives rise to forces tending to disrupt it, and that for each body there is a speed beyond which it will fly to pieces. Now for a planet, such as the earth, it has been calculated that when it rotated about its axis in a period of something like 4 hours, to which period we have traced our earth when the moon was close to it, the disruptive forces might be just about strong enough to cause it to fly to pieces. In such an event is it not possible that the moon was one of the pieces into which the rapidly rotating sphere had broken up?

But even had the attractive forces of the earth at that early time been sufficiently strong to withstand the disruptive forces due to rapid rotation, there were two other factors which, coöperating with these disruptive forces,

may have brought about the break-up of the rapidly rotating planet. These factors were the period of oscillation of the earth and the tide caused by the sun.

It will be recalled that in our consideration of the stationary-wave type of tidal movement, it was found that any given body of water has a definite period of oscillation which can be calculated from its dimensions. Furthermore, these oscillations become greater, the more nearly the period of the force giving rise to the oscillation approximates the period of the oscillations. Now calculation shows that at the time of which we are speaking, the earth's period of oscillation was somewhere in the neighborhood of two hours.

But the earth at that time rotated in a period of about 4 hours. The tide brought about by the sun in the molten mass of the earth therefore had a period of about 2 hours. Hence the natural period of the oscillation of the earth at that time coincided with the period of the solar tide, and as a consequence the two reinforced each other. The oscillations must therefore have become so violent that, in coöperation with the disruptive forces brought about by the rapid rotation, the planet might well have shaken to pieces, the moon being one or more fragments into which the planet had broken up.

Suppose, however, that at the time under consideration the earth, instead of being a molten mass, had a solid crust. Even in this event, the forces involved—the oscillation of the earth reinforced by the synchronization of the period of the tidal forces of the sun—must have been so enormous that the cohesion of a thin crust could not seriously resist them. And on the assumption of a solid crust at that time the hypothesis has been advanced that the Pacific Ocean is the hollow left at the place where the material forming the moon broke off from the earth.

Thus, the effects of tidal friction, followed back to the

remote past, furnish the bases of a theory that throws an interesting light on the early history of the earth and that affords an explanation of the circumstances under which the moon may have been born. To be sure there is considerable speculation involved in this, but it is speculation of a scientific character and its assumptions do not run counter to the orderly processes manifested in the universe.

The moon's period of rotation about its axis is now 27⅓ days—exactly the same as its period of revolution about the earth. But in the dim past when moon and earth were near each other and when the period of rotation of the earth was only a few hours, the moon, too, must have been rotating rapidly on its axis. The earth would then have raised on the moon enormous tides—larger tides by far than the very considerable ones raised on the earth by the moon at that time, for the mass of the earth is 80 times that of the moon. The moon's rotation was therefore impeded by the friction of these enormous tides, and it is to be noted that the effects of this friction were very much greater than the corresponding effects on the earth. In the first place the tides were much larger than on the earth; and in the second place, in consequence of the moon being a smaller body, the stock of rotational energy was much less. The slowing down of the moon's rotation to its present very leisurely period of 27⅓ days thus finds grounds for explanation in tidal friction.

The exact equivalence between the period of the moon's rotation and its period of revolution about the earth, which is a very striking fact, is a direct consequence of the theory of tidal evolution of earth and moon. For it is to be noted that the influence of tidal friction is not directed toward the total extinction of the rotation of the bodies affected by it. It is directed rather to reducing the speed of rotation to such a point that each body becomes tideless; that is, to the point that each body rotates on its axis in

the same period in which it revolves around the tide-producing body. In the case of the moon, because of the large tides that must have been rising and falling and because, too, of the relatively small mass as compared to the earth, this condition has already been reached.

The effects of the tidal friction are still continuing. The moon is getting further and further from the earth and the day and month are both lengthening, the day, however, at a much more rapid rate than the month. From these facts, what consequences may we deduce for the future? If we inquire of the mathematician he tells us that, basing his calculations on the theory of tidal friction, the gradual change in the lengths of the day and month will continue until they become equal, each at that time having a period of 55 of our present days. During all this time the tides on the earth will be decreasing because of the increasing distance of the moon, and when month and day become equal the lunar tide will completely cease, since the earth then will no longer rotate relatively to the moon, the two bodies then moving as though united by a rigid bar.

To the question as to how far in the future that time lies, the mathematician hesitates to answer, for all the factors involved are not known with sufficient precision. But from the fact that the lengthening of the day is estimated to be now at the rate of about one minute in six million years, it is obvious that the time in question lies in the very distant future and that it is to be counted in millions of years. And if one desires to multiply this a thousandfold or even more, there is nothing to prove he has overestimated the time.

The tides thus offer an explanation of the evolution of the earth and moon; and in the theory as put forward by Sir George Darwin a number of the features of the earth-moon system receive a rational and connected explanation. In the present chapter only a very meager outline of this

interesting phase of the subject could be given. Indeed, throughout this volume the limitation of space has precluded anything but the merest outline of the subject of tides, and has necessitated the exclusion of a number of interesting collateral problems. It is hoped, however, that the subject matter as presented is sufficient to indicate something of the nature and scope of tides and tidal phenomena. Prior to the formulation of the law of gravitation the study of the tides had engaged the attention of leading philosophers, and since Newton's time many distinguished mathematicians have contributed to its development. It still constitutes a fertile field for research, offering to the investigator a number of interesting problems.

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